DENSI TY ENHANCEMENT IN HELICON PLASMA SOURCES FOR OPERATIONAL FREQUENCIES NEAR THE LOWER HYBRID FREQUENCY

Mikhailenko V.S., Stepanov K.N.
Kharkov National University
Kharkov, Ukraine

Scime E.E.
West Virginia University
Morgantown, WV

1. The measurements in HELIX using a Nagoya III antenna showed clear evidence of monotonically increasing plasma density with decreasing RF frequency [Keiter, Scime, and Balkey, Phys. Plasmas, 4, 2741 (1997)]

2. The magnetic field and the RF frequency dependence of the electron density clearly indicate a correlation between density production and the lower hybrid frequency [Boswell R.W., Plasma Phys. Controll. Fusion 26, 1147 (1984); Cho S. Phys. Plasmas 7, 417, (2000)]. The majority of the highest density values lay in the range $0.5\hat{u}_{LH} < \hat{u}_0 < \hat{u}_{LH}$.

3. The experiments in HELIX have shown that, unless the source is operated at high pressure, the ion temperature in this source is highly anisotropic with $T_{\|} > T_{\perp}$ [Scime E.E., Keiter P.A., Zintl M.W., Balkey M.M., Kline J.L., and Koepke M.E., Plasma Sources Sci. and Technol., 7, 186 (1998)]

4. The perpendicular ion temperature in downstream region peaks for $\hat{u}_0 \sim 0.6\hat{u}_{LH}$. For $\hat{u}_0 > \hat{u}_{LH}$ the perpendicular ion temperature is relatively constant along the axis of the source. For $\hat{u}_0 < \hat{u}_{LH}$, an additional ion heating mechanism adds energy to the edge layer of plasma [Balkey M.M., R. Boivin, Kline J.L., Scime E.E., Plasma Sources Sci. and Technol. 10, 284 (2001)].

Collisional damping of helicon waves, particularly at low neutral pressures, as well as electron Landau damping, was insufficient to explain the high RF absorption efficiency of helicon waves. The parametric excitation of fast ion-sound waves, $\hat{u} = k\nu_s$, by the helicon pumping wave has been suggested as a possible mechanism for the anomalous absorption of the helicon wave and electron heating [A.I. Akhiezer, V.S. Mikhailenko, K.N. Stepanov, Phys. Lett. A 245, 117 (1998)].
Short-Wavelength Ion-Sound Parametric Turbulence of Plasma in Magnetic Field (case $k\tilde{n}_e >> 1$).

The kinetic parametric ion-sound instability is excited in non-isothermal plasmas with hot electrons and cold ions ($T_e >> T_i$) when the amplitude of the electrons’ oscillatory velocity $u$ in the field $E_0 = e_z E_0 x \cos(\omega_0 t - k_x r) + e_z E_0 y \sin(\omega_0 t - k_x r)$ of the helicon wave exceeds the ion-sound velocity $v_s$. The frequency $\omega(k)$ and growth rate $\gamma(k)$ of this instability in the case of $\omega_0 >> \sqrt{\omega_c u_{ce}}$ are [A.I. Akhiezer, V.S. Mikhailenko, K.N. Stepanov, Phys. Lett. A 245, 117 (1998)].

\begin{equation}
\omega(k) = kv_s \left(1 + k^2 r_{De}^2\right)^{-1/2},
\end{equation}

\begin{equation}
\gamma(k) = \gamma_s(k) = \sum_{p=-\infty}^{\infty} \gamma_{\rho_p}(k) = -\frac{\omega_0(k)}{2^{3/2} k_{\perp} p_e (1 + k^2 r_{De}^2)} \sum_{p=-\infty}^{\infty} J_p^2(a) z_{\epsilon_0}^{(p)} \exp\left(-\left(z_{\epsilon_0}^{(p)}\right)^2\right),
\end{equation}

where $z_{\epsilon_0}^{(p)} = (\omega_0(k) - p\omega_0) / \sqrt{2|k_{\perp}|} \nu_{\epsilon_e}, \quad a = a_{\epsilon_e} = c \left(k^2 E_{0x}^2 + k_{0y} E_{0z}^2\right)^{1/2} / \omega_0 B_0$, $k_{\perp} p_e >> 1$, $k_{\parallel} p = k_{\parallel} - p k_{0\parallel}$. In the case of strong ellipticity of the pumping wave, when $E_{0x} > E_{0y}$, the growth rate is at maximum with

\begin{equation}
\gamma_{max} \sim 0.05 (\omega_c \omega_{ci})^{1/2} \left(1 + k^2 r_{De}^2\right)^{3/2}.
\end{equation}

The ions are treated as unmagnetized ($2\pi a_{\epsilon_e} >> \omega_{ci}, k_{\perp} p_e >> 1$). For this instability, the wavelength of unstable ion-sound oscillations $1/k$ is considerably less than the electron Larmor radius, $k \rho_e > 1$. As these oscillations are excited at the pumping frequency $\omega_0 > \omega/m$ ($m$ is an integer) and $a_{\epsilon_e} = k_{0\parallel} \omega_0 > 1$, then the condition $k \rho_e >> 1$ holds for $\omega_0 >> \sqrt{\omega_{ce} \omega_{ci}}, \quad m \sim 1$. The same condition holds in a number of experiments by F.F. Chen. [Phys. Plasmas 3, 1783 (1996)] for which the

The bulk ions are non-resonant and induced scattering of ions by ion-sound waves results in ion heating. The quasilinear equation for the average ion distribution function $F_{i0}$ in which induced scattering of ion-sound waves on ions is included is

$$\frac{\partial F_{i0}}{\partial t} = \frac{\pi e^2}{m_i^2} \int dk \left( k \cdot \frac{\partial}{\partial v_i} \right) I(k) \delta(\omega(k) - k \cdot v_i) \left( k \cdot \frac{\partial F_i}{\partial v_i} \right) +$$

$$\frac{\pi}{m_i^2 n_{i0}^2} \int dk \int dk' I(k) I(k') \left( \frac{\partial \epsilon}{\partial \omega(k)} \right)^{-1} \left( \frac{\partial \epsilon}{\partial \omega(k')} \right)^{-1} \left( k - k' \right) \cdot \frac{\partial}{\partial v_i} \left( k \cdot v_i \right) \times$$

$$\frac{(k \cdot k')^2}{k^2 k'^2} \delta(\omega_i(k) - \omega_i(k') - (k - k') \cdot v_i)(k - k') \cdot \frac{\partial F_{i0}}{\partial v_i}.$$

(4)

The wave vectors, $k$, of the ion-sound waves excited by the kinetic parametric instability are directed almost across magnetic field. Thus, we obtain from (4) that through this process, the ions are heated predominantly across the magnetic field. The ion-heating rate is

$$\frac{\partial T_{i\perp}}{\partial t} = \frac{T_i}{\tau_{i\perp}}$$

(5)

where
\[
\frac{1}{\tau_{i\perp}} = \frac{T_i}{2m^2 n_0^2} \int dk \int dk' I(k) I(k') \left( \frac{\partial \epsilon}{\partial \omega(k)} \right)^{-1} \left( \frac{\partial \epsilon}{\partial \omega(k')} \right)^{-1} \frac{(k \cdot k')^2}{k^2 k'^2} \times \nonumber \\
(k \times k')^2 \delta(\omega, k - \omega', k'))
\] (6)

and ion temperature anisotropy with \( T_{i\perp} > T_{i\parallel} \) arises. This anisotropy is not destroyed by ion-ion collisions when \( \tau_{i\perp} v_{ii} < 1 \). This condition is satisfied in the WVU and UCLA experiments [E.E. Scime, P.A. Keiter, M.W. Zintl, M.M. Balkey, J.L. Kline, and M.E. Koepke, Plasma Sources Sci. Technol. 7, 186 (1998); F.F. Chen, I.D. Sudit, M. Light, Plasma Sources Sci. Technol. 5, 173 (1996)].

When the saturated state is attained due to the induced scattering of ion-sound waves on ions [A.I. Akhiezer, V.S. Mikhailenko, K.N. Stepanov, Phys. Lett. A 245, 117 (1998)],

\[
\frac{1}{\tau_{i\perp}} = \int dk \gamma(k) \frac{W(k)}{n_e T_e} - \frac{1}{\tau_{i\parallel}}
\] (7)

and

\[
\frac{\partial T_{i\perp}}{\partial t} \sim \frac{\partial T_e}{\partial t} = \frac{T_e}{\tau_{e\parallel}} \sim \frac{\gamma_e W}{n_e e_0}.
\] (8)

Therefore: **anisotropic heating of ions is an inherent property of turbulent heating of ions in helicon plasma sources.**

It is interesting to note that the growth rate (2), and therefore the electron and ion heating rates (8), are maximal, when \( k_0 \sim 1 \) and the parameter \( a \sim 1 \). With the condition \( \dot{u}_s = m \dot{u}_0 \), which is the necessary condition for the excitation of the kinetic parametric instability, all these conditions fulfilled when \( \dot{u}_0 \approx \dot{u}_{\text{LH}} \).
Long-Wavelength Ion-Sound Parametric Turbulence of Plasma in Magnetic Field (case $k\rho_e \sim 1$)

There are a number of helicon source experiments for which the magnetic field is “strong”, $\omega_0 \gtrsim \sqrt{\omega_{ce}\omega_{ci}}$. In this case it is necessary to consider the long wavelength, $k\rho_e \lesssim 1$, ion-sound instability. Given a plasma immersed in the electric field of a pump wave (helicon) of frequency $\omega_0$ that is considerably below the electron cyclotron frequency $\omega_{ce}$ and considerably more than the ion cyclotron frequency $\omega_{ci}$, for $u \gtrsim v_s$ ion-sound oscillations may be excited for which nonuniformities in the plasma and pumping field in the direction perpendicular to the magnetic field can be neglected. For excited oscillations that propagate almost across the magnetic field, the longitudinal wavenumbers of the unstable oscillations $k_\parallel$ and the helicon wave $k_\parallel 0$ may be of the same order of magnitude, generally speaking. Therefore, we must take into account the nonuniformity of the helicon electric field along the magnetic field.

We describe the linear stage of the ion-sound parametric instability with the following infinite set of difference equations for the Fourier components of the electric potential of the ion-sound wave determined in the reference frame oscillating together with ions in the pumping wave field [Kitsenko A.B., Panchenko V.I., Stepanov K.N., Tarasenko F.F., “Parametric instabilities and turbulent heating of a plasma in the field of a fast magneto-acoustic wave,” *Nuclear Fusion* 23, 527 (1973); Korzh A.F., Mikhailenko V.S., Stepanov K.N., “Parametric Ion Cyclotron Instability of Plasma in the Electric and Magnetic Fields of the Nonuniform MHD wave,” *Sov. Plasma Physics* 15, 413 (1989)]
\[
0=(1+\delta\varepsilon_i(\omega,\mathbf{k}))\varphi(\omega,\mathbf{k}) + \sum_{n=-\infty}^{\infty} \sum_{p=-\infty}^{\infty} J_p(a_E)J_{p+n}(a_E)e^{i\delta}. 
\]

where \(\delta\varepsilon_\alpha\) is the contribution to the longitudinal dielectric permittivity made by \(\alpha\)-species particles. For the oscillations considered here, the growth rate exceeds the ion cyclotron frequency and the wavelength is much less than the ion Larmor radius. Thus, we can neglect the action of the magnetic field on ion motion, so that

\[
\delta\varepsilon_i(\omega,\mathbf{k}) = \frac{\omega_{pi}^2}{k^2v_{Ti}^2} \left[ 1 + i\sqrt{\pi} z_i W(z_i) \right]. 
\]

For electrons, neglecting the small terms proportional to \(\omega^2/\omega_{ce}^2\) and \((k_||v_{Te}^2)/\omega_{ce}^2\), we obtain

\[
\delta\varepsilon_e(\omega,\mathbf{k}) = \frac{\omega_{pe}^2}{k^2v_{Te}^2} \left[ 1 + i\sqrt{\pi} A_0\left(k^2\rho_e^2\right) z_e W(z_e) \right]. 
\]

Here the following notation is introduced: \(\omega_{p\alpha} = \sqrt{4\pi n_0\epsilon_\alpha^2/m_\alpha}\) is the Langmuir frequency of \(\alpha\)-species particles, \(v_{T\alpha} = \sqrt{T_\alpha/m_\alpha}\) is their thermal velocity, \(z_i = \omega/\left(\sqrt{2}|k|v_{Ti}\right)\), \(z_e = \omega/\left(\sqrt{2}|k_||v_{Te}\right)\),

\[
W(z) = \left[ 1 + \frac{2i}{\sqrt{\pi}} \int_0^z e^t dt \right]. 
\]
Making in Eq. (9) the substitution $\omega \to \omega - m\omega_0$, $k_\parallel \to k_\parallel - mk_\parallel$, we obtain the set of difference equations, whose solubility criterion is the vanishing of the determinant

$$D = \det \left| a_{mn} \right|_{m,n=-\infty}^\infty = 0,$$

where

$$a_{mn} = \delta_{mn} + \frac{\exp\left[-i(n-m)(\delta + \pi)\right]}{1 + \delta\epsilon \left(\omega - m\omega_0, k\right)} \sum_{p=-\infty}^\infty J_{p+m}(a_E) \cdot J_{p+n}(a_E) \delta\epsilon \left(\omega + p\omega_0, k + p k_\parallel\right).$$

For short wavelength oscillations with $k\rho_e >> 1$, Eq. (12) requires the vanishing of any diagonal element, i.e.,

$$D - 1 + \frac{1}{1 + \delta\epsilon \left(\omega - m\omega_0, k\right)} \sum_{p=-\infty}^\infty J_{p+m}^2(a_E) \cdot \delta\epsilon \left(\omega + p\omega_0, k + p k_\parallel\right) = 0.$$

This equation has been studied before for $k_\parallel = 0$ [Kitsenko A.B., Panchenko V.I., Stepanov K.N., Tarasenko F.F., “Parametric instabilities and turbulent heating of a plasma in the field of a fast magneto-acoustic wave,” Nuclear Fusion 23, 527 (1973)] and for $k_\parallel \neq 0$ [A.I. Akhiezer, V.S. Mikhailenko, K.N. Stepanov, Phys. Lett. A 245, 117 (1998)]. Here we have Eq. (13) for $k\rho_e << 1$ and pumping frequencies $\omega_0 >> \sqrt{\omega_{ce}\omega_{ci}}$. 
Numerical solution of the dispersion equation for long wavelength oscillations

On solving Eq. (13) numerically, we assumed \( \omega_0/\sqrt{\omega_{ce} \omega_{ci}} = 0.6, \omega_{pe}/\omega_{ce} = 25 \) (helicon branch of oscillations is usually treated under the condition \( \omega_{pe}^2 >> \omega_{ce}^2 \)). \( T_e/T_i = 20, \ u/v_s = 5 \), the operating gas is hydrogen. Conventionally one uses argon in helicon sources as an operating gas. It is easy to prove that under the condition \( kr_{De} << 1 \) the result of calculation does not depend on the mass of the operating gas if the quantities \( \omega_0/\sqrt{\omega_{ce} \omega_{ci}} \) and \( u/v_s \) are regarded as fixed.

Figure 1 shows the dependence of the oscillation frequency on the wave vector. For comparison, the same figure shows the dependence of the ion-sound frequency \( \omega = kv_s \). It is seen that the frequency depends linearly on the wave vector. The difference between the frequency of unstable oscillations and the frequency of ion-sound oscillations in an unmagnetized plasma is associated with the effects of finite electron Larmor radius and the presence of the pumping wave.

Figure 2 shows the growth rate for different values of the angle \( \theta \) between the magnetic field and the wave vector.
Fig. 1. Frequency of parametrically unstable ion-sound oscillations in the helicon field versus wavenumber ($\omega^2/\omega_{ce}^2 = 25$, $v/v_s = 5$, $\tau_e/\tau_i = 20$, $\omega_e/\sqrt{\omega_{ce} \omega_{ci}} = 0.6$, $k_{||0} = 0$).

Fig. 2. Growth rate of unstable ion-sound oscillations versus wavenumber for different $\cos\theta$ values (the parameters are the same as in Fig. 1). The curves correspond to $\cos\theta = 0.01$ to 0.05 in steps of 0.005.
We see that for increasing $\cos \theta = k_\parallel / k$, the value of the growth rate peaks at $\cos \theta = 0.045$. The maximum value of the growth rate is $\gamma_{\text{max}} \approx 0.3 \sqrt{\omega_{ce} \omega_{ci}}$. At larger $\cos \theta$ values, the maximum value of the growth rate decreases. The largest growth rate is achieved at $k \rho_e \approx 0.75$. In this case the frequency is $\omega \approx 0.9 \sqrt{\omega_{ce} \omega_{ci}}$ and the excitation of oscillations is due to the beats with $p=2 \ (p \omega_0 > \omega)$. The maximum value of the growth rate in Fig. 2 is related to the value $|z_{ep}| \sim 1$. In this case, the interaction of resonant electrons with velocities of the order of the thermal velocity with the $p$-th beat waves appears to be important.

These calculations demonstrate that in the case of “strong” magnetic fields, the ion-sound parametric instability in the helicon field also occurs. The growth rate of the ion-sound parametric instability is even stronger than in the case of “weak” magnetic field, for which $\gamma_{\text{max}} \sim 0.05 \sqrt{\omega_{ce} \omega_{ci}}$. Therefore, one should also expect in the “strong” magnetic field case the appearance of a strong ion-sound turbulence leading to electron heating and discharge sustainment.
Ion Landau heating in strong magnetic field (or low frequency pumping wave)

When the frequency of pumping wave is \( \dot{u}_0 = r_0 \dot{u}_{LH} < \dot{u}_{LH} \) (where \( r \leq 1 \)), we find that the wavelength of the Trievel-piece-Gould mode may be obtained from the equation \( \dot{u}_0 = r_0 \sqrt{u_{ce} \dot{u}_{ci}} = u_{ce} \cos \theta \), or \( k_\perp = r_0^{-1} k_\parallel \sqrt{m_i / m_e} \). Then,

\[
\frac{\dot{u}_0}{\sqrt{2k_\parallel v_{T_e}}} = \frac{1}{\sqrt{2k_{\perp} v_{Te}}} \quad \text{and} \quad \frac{\dot{u}_0}{\sqrt{2k_{\parallel} v_{Te}}} = \frac{r_0}{\sqrt{2k_{\perp} v_{Te}}} \sqrt{\frac{T_e}{T_i}}.
\]

In the case of \( \frac{r_0}{\sqrt{2k_{\perp} v_{Te}}} \sqrt{\frac{T_e}{T_i}} \sim 1 \),

we have strong resonance heating of ions in the fields of the TG mode, i.e. surface heating.