PHYS783 - Plasma Kinetic Theory
Assignment # 1
Due January 21, 2009 (Can turn in Jan. 26 if you are taking the Mechanics Qual)

Reminders: Show your work! Include references on your submitted version. Write legibly!

1. **One-dimensional simple harmonic oscillator in phase space**
   Consider a mass m attached to a massless spring with spring constant k.
   (a) Using either Hamiltonian or Newtonian mechanics, write down the equation of motion for the mass m.
   (b) Solve for the velocity v as a function of the position x, allowing arbitrary initial conditions x_0 and v_0.
   (c) Draw the phase space trajectory for a family of initial conditions. Be sure to put an arrow on them so you can tell which direction the particle is moving. Also label your plot (including axes and intercepts) appropriately. What shape are the trajectories in phase space?
   (d) You should find trajectories where the masses have trapped orbits. Do you find any with passing orbits?
   (e) Now, suppose a weak drag force is introduced to the system (the mass-spring system is immersed in a gas or fluid). The drag force goes like $F = -bv$ where b is a small constant (*i.e.,* the oscillator is underdamped). Draw a representative phase space trajectory for the mass in this case. Try doing so *without* doing a full calculation.

2. **One-dimensional collisions in phase space**
   Consider a particle of mass m. The particle travels in from $x = -\infty$ at a speed $v_0 > 0$. At $x = 4$ sits a hard wall. The mass strikes the wall and turns around.
   (a) Plot the mass' trajectory in phase space under the assumption that (1) the collision is perfectly elastic, and (2) the collision occurs over an infinitely small time.
   (b) What property do you notice about the trajectory in phase space under these circumstances? For example, contrast the trajectory to other trajectories we have discussed. (Your result to this problem will be very important later in the course!)
   (c) Now suppose that the collision is still elastic, but takes place over a finite time. Make a schematic diagram showing the phase space trajectory of the mass if the collision time is small but finite.

3. **Statistics**
   Suppose you are given that a distribution function is
   $$f(v) = \frac{a}{v^2 + \gamma^2},$$
   where a and γ are constants. There is no x dependence.
   (a) State, in words, what this means.
   (b) Relate a to γ and the number density of particles n and plot $f(v)$.
   (c) What is the probability of finding a particle with velocity v between 2γ and 2.2γ?
   (d) What is the *probability density* of finding a particle with a velocity between 2γ and $2\gamma + dv$, where dv is infinitesimally small?
   (e) Extra credit - What is the probability of finding a particle with velocity exactly equal to 2γ?