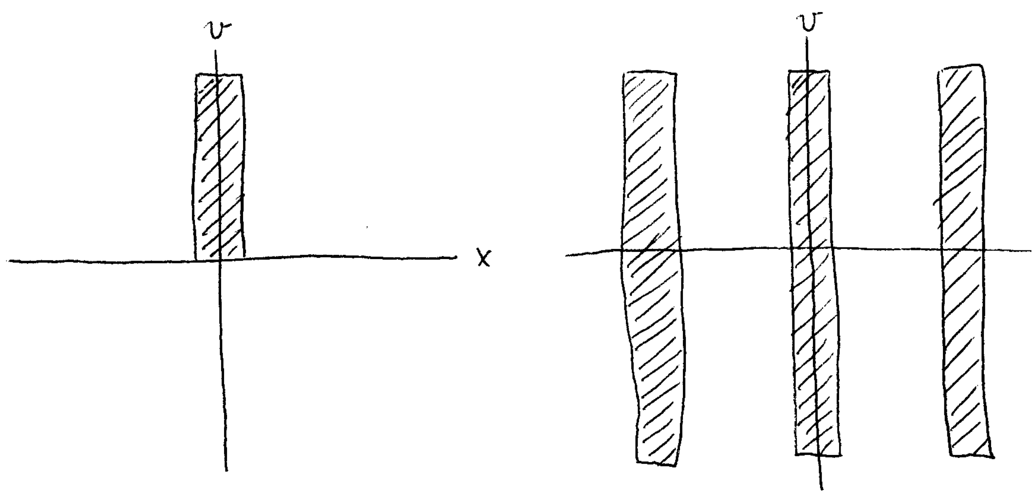
Phys 783 - Plasma Kinetic Theory
Assignment # 3
Due Friday, February 13, 2009

Reminders: Show your work! Include references on your submitted version. Write legibly!

1. **Distribution Function Evolution**

   At $t = 0$, a system is described by the distribution function in the figure below on the left. Assuming there are no forces on the particles, sketch the distribution function an instant in time later. Repeat this exercise for the distribution function on the right.



2. **Bulk Properties of Vlasov Equation Solutions**

   A cylindrically symmetric plasma has a distribution function f given in terms of the Hamiltonian H and canonical angular momentum $P_\theta = r(mv_\theta + qA_\theta/c)$ as

$$f(\mathbf{r}, \mathbf{v}) = \frac{n_0}{(2\pi T/m)^{3/2}} \exp\left(-\frac{H - \lambda P_\theta}{T}\right),$$

   where n_0, λ, and T are constants and the vector potential $A_\theta(r)$ produces a uniform magnetic field in the z direction. We know this solves Vlasov's equation because H and P_θ are constants of the motion. Find expressions for the density and rotation rate as a function of r.

3. **Solutions of Vlasov Equation**

   We will show, using two different techniques, that $f(X, Y, v_\perp^2, v_z)$ is a solution of the Vlasov equation for a system with a uniform magnetic field $\mathbf{B} = B_0\hat{\mathbf{z}}$, where $X = x + v_y/\Omega, Y = y - v_x/\Omega, v_\perp^2 = v_x^2 + v_y^2$, and $\Omega = qB_0/mc$.

   (a) Technique I - Do a direct calculation.

   (b) Technique II - Show from the equations of motion that all the arguments are constants of the motion. What are the physical interpretations of X and Y?

4. **More Solutions of Vlasov Equation**

Consider a distribution function of the form

$$f(\mathbf{r}, \mathbf{v}, t) = n(\mathbf{r}, t)\delta(\mathbf{v} - \mathbf{u}(\mathbf{r}, t)),$$

where $\delta(\mathbf{v} - \mathbf{u}(\mathbf{r}, t))$ is a three dimensional delta function. Determine the conditions on n and $\mathbf{u}$ that make this distribution function a solution to the Vlasov equation. This problem will test your math abilities! You may find the following identity useful:

$$g(x)\frac{d}{dx}\delta(x - x_0) = g(x_0)\frac{d}{dx}\delta(x - x_0) - \delta(x - x_0)\frac{dg(x_0)}{dx}.$$