Phys 783 - Plasma Kinetic Theory
Assignment # 5
Due Wednesday, March 4, 2009

Reminders: Show your work! Include references on your submitted version. Write legibly!

1. **Landau Damping for a Random Velocity Distribution**

   Consider the velocity distribution function given by

$$F_{e0}(v_z) = \begin{cases} n_0/2v_{th} & |v| \leq v_{th} \\ 0 & \text{otherwise} \end{cases}$$

   where v_{th} is a typical velocity scale in the plasma, n_0 is a constant, and $F_{e0} = \int d^2v_\perp f_{e0}$. Do you expect Landau damping of electron plasma waves to occur in this system? Confirm or refute your prediction by calculating the dielectric function $\epsilon(\mathbf{k}, \omega)$ using

$$\epsilon(\mathbf{k}, \omega) = 1 + \frac{4\pi e^2}{k^2 m_e} \int d^3v \frac{\mathbf{k} \cdot \nabla_v f_{e0}}{\omega - \mathbf{k} \cdot \mathbf{v}} = 1 + \frac{4\pi e^2}{k^2 m_e} \int dv_z \frac{k}{\omega - kv_z} \frac{\partial F_{e0}}{\partial v_z}, \tag{1}$$

   then finding the dispersion relation $\omega(\mathbf{k})$ by finding the roots of $\epsilon = 0$. (Hint - the integral above can be carried out using standard techniques, *i.e.,* it does not require contour integration.) Comment specifically about resonant particles.

2. **Landau Damping Rate for Lorentz Distribution**

   When we derived Landau damping of electron plasma waves in class, we found that the velocity space integral for the dielectric function [see Eq. (1) above] for a Maxwellian distribution is difficult to perform since one cannot close the contour at infinity. However, for a Lorentzian distribution,

$$F_{e0}(v_z) = \frac{n_0}{\pi} \frac{v_{th}}{v_z^2 + v_{th}^2}$$

   where v_{th} is a typical velocity and n_0 is a constant, the contour can be closed and the integral is much more tractable.

   (a) Evaluate the damping rate for the Lorentzian initial distribution function, showing that electron plasma waves damp as $\exp(-kv_{th}t)$, *i.e.,* the damping rate is $\omega_i = -kv_{th}$. Hint - You may find the mathematics to be a little easier if you use the form of ϵ which has been integrated by parts.

   (b) For a Maxwellian distribution, we showed in class that the Landau damping rate is $\omega_i \simeq -(\pi/8)^{1/2}\omega_{pe}(k\lambda_{De})^{-3} \exp[-(1 + 3k^2\lambda_{De}^2)/2k^2\lambda_{De}^2]$. Assuming $v_{th}^2 = 2T_{e0}/m_e$ for the Lorentzian distribution, show that the Lorentzian damping rate is larger than the Maxwellian damping rate (use whatever technique you would like, but a plot would be sufficient). Qualitatively, why would you expect the Landau damping to be stronger for the Lorentzian than the Maxwellian distribution? (Hint - Consider plotting the two distributions.)

3. **Kinetic Theory of Ion Acoustic Waves**

In the previous homework, we used fluid theory to derive the properties of ion acoustic waves. In this problem, we will treat ion acoustic waves using kinetic theory, thereby capturing the effect of Landau damping.

(a) Calculate the (approximate) frequency of oscillations (ω_r, the real part of ω). You can assume from the outset that both electrons and ions initially have Maxwellian distribution functions. Also, recall from the previous homework that the appropriate parameter regime for ion acoustic waves is $kv_{th,i} \ll \omega \ll kv_{th,e}$. To find ω_r, start from the dielectric function

$$\epsilon(\mathbf{k}, \omega) = 1 + \sum_s \frac{4\pi q_s^2}{k^2 m_s} \int d^3v \frac{\mathbf{k} \cdot \nabla_v f_{s0}}{\omega - \mathbf{k} \cdot \mathbf{v}}$$

and calculate the real part by splitting the integral into a principal value part and a residue part (as we did in class). Then, set it equal to zero to find the real frequency of oscillations ω_r. To make a proper comparison, keep the lowest order thermal corrections in the ion term (but not the electron term) and confirm that you obtain the same result you got from the fluid theory.

(b) Calculate the damping rate for ion acoustic waves ω_i. Again assume Maxwellian distributions for both electrons and ions, and keep the contributions from both the electrons and the ions. Show that the damping rate ω_i is approximately

$$\omega_i = -\frac{\omega_r^4}{k^3 \omega_{pi}^2} \sqrt{\frac{\pi}{8}} \left[\omega_{pe}^2 \left(\frac{m_e}{T_{e0}} \right)^{3/2} e^{-m_e \omega_r^2 / 2k^2 T_{e0}} + \omega_{pi}^2 \left(\frac{m_i}{T_{i0}} \right)^{3/2} e^{-m_i \omega_r^2 / 2k^2 T_{i0}} \right].$$

Show that in the appropriate limit, this reduces to

$$\omega_i = -\frac{\omega_r^4}{k^3 c_s^3} \sqrt{\frac{\pi}{8}} \left[\sqrt{\frac{m_e}{m_i}} + \left(\frac{T_e}{T_i} \right)^{3/2} \exp\left(-\frac{T_e/2T_i}{1 + k^2 \lambda_{De}^2} - \frac{3}{2} \right) \right].$$

Which term (corresponding to which species) plays the dominant role? Note, this shows that ion acoustic waves can only propagate without appreciable damping if $T_e \gg T_i$.