Phys 783 - Plasma Kinetic Theory
Assignment # 8
Due Wednesday, April 8, 2009

Reminders: Show your work! Include references on your submitted version. Write legibly!

1. **Two Stream Instability** (Essentially parts of Bellan, Problem 5.1)

   (a) Use fluid theory to show that one gets the same result for the dielectric function ϵ for the *cold plasma* two stream instability as the kinetic theory result we found in class, namely

   $$\epsilon(\mathbf{k},\omega) = 1 - \frac{\omega_{pi}^2}{\omega^2} - \frac{\omega_{pe}^2}{(\omega - \mathbf{k}\cdot\mathbf{u})^2}.$$

   Assume the equilibrium is made up of uniform electron and ion densities with no electric field, with the electrons having an equilibrium bulk flow of $\mathbf{u}_{e0} = \mathbf{u}$ and the ions having no equilibrium bulk flow. (Hint - To jog your memory, it might help to look at your solution to problem 2 on Homework 4. The analysis here is similar, but you're taking cold ions and electrons and the electrons have a non-zero equilibrium flow.)

   (b) Assume, for simplicity, that $\mathbf{k}$ is parallel to $\mathbf{u}$. Letting

   $$g(\omega) = \frac{\omega_{pi}^2}{\omega^2} + \frac{\omega_{pe}^2}{(\omega - ku)^2}.$$

   show that the condition for instability, $\min(g) > 1$, is met when

   $$|ku| < \omega_{pe}\left[1 + \left(\frac{m_e}{m_i}\right)^{1/3}\right]^{3/2} = \left(\omega_{pe}^{2/3} + \omega_{pi}^{2/3}\right)^{3/2}.$$

   Hint - Let ω_* be the value of ω where $g(\omega)$ is a minimum. Find ω_* and evaluate $g(\omega_*)$. It might surprise you that the relative speed must be <u>small</u> enough to have an instability! Make sure it makes sense to you why (physically) this is true!

   (c) One can show - *you're not being asked to!* - that the fastest growing mode for the cold plasma two stream instability satisfies $ku \simeq \omega_{pe}$. Show that the growth rate $\gamma = \omega_i$ of the mode with ku exactly equal to ω_{pe} is approximately

   $$\gamma = \frac{\sqrt{3}}{2}\left(\frac{\omega_{pe}\omega_{pi}^2}{2}\right)^{1/3}$$

   Hint - Avoid a difficult calculation for an exact solution by determining an appropriate parameter regime for the solution. In particular, convince yourself that there is a solution if $\omega^2 \ll \omega_{pe}^2$ and do an expansion.

   (d) In general, why is it important to know about the fastest growing mode?

2. **Landau Damping of Ion Acoustic Waves vs. Two Stream Instability**
(Essentially Bellan, Problem 5.8)

(a) Consider a plasma with a warm ion population with no mean velocity and a warm electron population with mean speed $\mathbf{u}$. As a first step, write down appropriate (one dimensional) distribution functions F_{i0} and F_{e0} for the two species assuming (drifting) Maxwellians. Then, starting from the general expression for the electrostatic dielectric function derived in class,

$$\epsilon(\mathbf{k}, \omega) = 1 + \frac{4\pi e^2}{k^2 m_e} \int dv_z \frac{k}{\omega - kv_z} \frac{\partial F_{e0}}{\partial v_z},$$

show that the dielectric function can be written as

$$\epsilon(\mathbf{k}, \omega) = 1 + \frac{1}{k^2 \lambda_{Di}^2} \left[1 + \frac{\omega}{kv_{th,i}} Z\left(\frac{\omega}{kv_{th,i}}\right)\right] + \frac{1}{k^2 \lambda_{De}^2} \left[1 + \left(\frac{\omega - ku}{kv_{th,e}}\right) Z\left(\frac{\omega - ku}{kv_{th,e}}\right)\right],$$

where we assume $\mathbf{k}$ is parallel to $\mathbf{u}$ and the thermal speeds are defined as $v_{th,s}^2 = 2T_{s0}/m_s$. *(Note - this definition differs than that on Homework 4 by a numerical factor!)*

(b) Now, we'll determine when there is damping or an instability. First, argue that the condition for instability is $\epsilon_i < 0$. Then, using $Im[Z(\xi)] \simeq \sqrt{\pi} e^{-\xi^2}$ and the appropriate parameter regime for ω and k, show that instability occurs when

$$\omega_r < ku.$$

Note that in the $T_{e0} \gg T_{i0}$ limit, which is required for the ion acoustic wave to propagate, this condition can be written as

$$u^2 > \frac{c_s^2}{1 + k^2 \lambda_{De}^2}$$

(c) Recall that we said that - in general - when there is an instability, there must be a source of free energy and a place that the free energy can go. Without doing a calculation, what is the source of free energy in this problem and where does the free energy go? Why is this result important? (People have referred to this effect as an "effective drag." Why, strictly speaking, is drag not a good word to describe it?)