Scaling of asymmetric Hall magnetic reconnection

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The scaling of the reconnection rate and ion and electron outflow speeds with upstream magnetic field strengths and plasma mass densities during asymmetric collisionless (Hall) reconnection without a guide field is studied using two-dimensional two-fluid simulations. The results agree with a recent theory by Cassak and Shay (2007). It is found that the normalized reconnection rate is on the order of 0.1 and is independent of the asymmetry in field or density. Signatures of asymmetric Hall reconnection and applications to the magnetopause are briefly discussed.

1. Introduction

Canonical models of magnetic reconnection [Sweet, 1958; Parker, 1957; Petschek, 1964] are usually studied in two dimensions with plasmas on either side of the dissipation region having identical densities and magnetic field strengths. However, these conditions are rarely realized in nature, most notably at the dayside magnetopause [Levy et al., 1964], where field strengths differ by a factor of 2–3 and densities differ by about an order of magnitude from the magnetosheath to the magnetosphere [Phan and Paschmann, 1996; Ku and Sibeck, 1997].

Until recently, little was known about the scaling of asymmetric reconnection, meaning the functional dependence of reconnection parameters on the upstream fields $B$ and densities $\rho$. Borovsky and Hesse [2007] used magnetohydrodynamic (MHD) simulations with localized resistivity to study the scaling for systems with asymmetric $\rho$. General scaling laws for two-dimensional anti-parallel asymmetric reconnection were derived by Cassak and Shay [2007]. These laws have been successful in comparisons to simulations, namely (collisional) Sweet-Parker reconnection with asymmetric $B$ [Cassak and Shay, 2007], MHD reconnection with anomalous resistivity with both $B$ and $\rho$ asymmetric [Birn et al., 2008], and global magnetospheric MHD simulations with an anomalous resistivity [Borovsky et al., 2008]. Birn et al. [2008] found that agreement is improved by including compression effects within the dissipation region.

These scaling studies were performed using a (uniform or localized) resistivity to break the frozen-in condition. However, reconnection allowed by classical collisions would never take place in the magnetosphere because it is sufficiently rarified. Scaling laws for collisionless systems are imperative for magnetospheric applications including observed asymmetries during reconnection in flux transfer events [Sanny et al., 1998], the distant magnetotail [Oberoslet et al., 2004], the solar wind [Gosling et al., 2006], and at the dayside magnetopause [Mozer et al., 2008], as well as the effect of plasmaspheric drainage plumes on dayside reconnection [Borovsky and Denton, 2006].

In this paper, we show that the scaling laws for the reconnection rate $E$ and outflow speed $v_{\text{out}}$ derived by Cassak and Shay [2007] are valid for two-dimensional anti-parallel Hall reconnection. Furthermore, the normalized reconnection rate $E'$ is independent of asymmetries in the asymptotic fields and densities, supporting the result that $E' \sim 0.1$ independent of system parameters [Shay et al., 1999; Huba and Rudakov, 2004; Shay et al., 2004]. These results extend similar work for MHD reconnection with an anomalous resistivity [Birn et al., 2008; Borovsky et al., 2008] to fast reconnection in a self-consistent system with the Hall effect.

2. Theory

The standard Sweet-Parker scaling laws were generalized to accommodate asymmetric conditions by Cassak and Shay [2007], with the scaling of $E$ and $v_{\text{out}}$ given by

$$E \sim \frac{B_1 B_2}{B_1 + B_2} \frac{v_{\text{out}}}{c} \frac{2 \delta}{L}$$

(1)

$$v_{\text{out}}^2 \sim \frac{B_1 B_2}{4 \pi} \frac{B_1 + B_2}{\rho_1 B_2 + \rho_2 B_1}$$

(2)

where “1”, “2”, and “out” refer to the upstream and outflow edges of the dissipation region and $\delta$ and $L$ are the half-width and half-length of the dissipation region. These expressions were derived from first principles using conservation laws assuming a two-dimensional system with no out-of-plane (guide) magnetic field. No dissipation mechanism was assumed, so it was claimed that these results hold for all types of reconnection, including Hall reconnection. See Swisdak and Drake [2007] for an alternate derivation of equation (2).

An implication of equation (1) is that the reconnection rate normalized to a field of $B_{\text{crit}} = 2 B_1 B_2 / (B_1 + B_2)$ and velocity of $v_{\text{out}}$ is equal to $\delta / L$, the aspect ratio of the ion dissipation region. The factor $\delta / L$ can, in principle, depend on $B$ and $\rho$, as it does in both symmetric [Parker, 1957] and asymmetric [Cassak and Shay, 2007] Sweet-Parker reconnection. We show in the next section that the normalized reconnection rate $E'$ is independent of $B$ and $\rho$ for asymmetric Hall reconnection.

In Hall reconnection, the ions decouple from the electrons and the magnetic field within one effective gyro-
radius of the neutral line. Since equation (2) was derived from conservation laws, we expect it to hold both for the ion outflow speed \(v_{\text{out}}\) and the electron outflow speed \(v_{\text{eout}}\). The appropriate fields and densities must be evaluated at the upstream edge of the ion (electron) dissipation region for the ions (electrons). The electron and ion upstream fields and densities are potentially significantly different.

3. Numerical Simulation Results

[8] The scaling of asymmetric Hall reconnection is investigated using the massively parallel two-fluid code F3D (described in detail in Shay et al. [2004]) to perform two-dimensional simulations. The density \(\rho\), ion velocity \(v_i\), ion pressure \(P_i\), and magnetic field \(B\) are evolved in time. The ions are assumed to be an adiabatic ideal gas with \(dP_i/\rho v_i^2/\gamma\) = 0, where \(\gamma = 5/3\) is the ratio of specific heats. The electron pressure is zero and is not evolved.

[10] The initial conditions have a magnetic field with a two-sided Harris sheet profile,

\[
B_x(z) = \begin{cases} 
-B_{01} \tanh\left(\frac{z - L_z/4}{w_0}\right) & L_z/4 < z < L_z/2 \\
-B_{02} \tanh\left(\frac{L_z - z}{4w_0}\right) & 0 < z < L_z/4.
\end{cases} 
\]

(3)

where \(w_0\) is the initial current sheet thickness, \(x\) is the direction of the reversed field, \(z\) is the inflow direction, \(L_z\) is the length of the domain along \(z\), and \(B_{01}\) and \(B_{02}\) are asymptotic field strengths. There is no initial guide field. The initial density profile is

\[
\rho(z) = \frac{1}{2} (\rho_{01} + \rho_{02}) + \frac{1}{2} (\rho_{01} - \rho_{02}) \tanh\left(\frac{z - L_z/4}{w_0}\right)
\]

(4)

for \(0 < z < L_z/2\), with asymptotic values of \(\rho_{01}\) and \(\rho_{02}\). The initial ion pressure enforces global pressure balance, \(P_i(z) + (B_x(z))^2/8\pi = \text{constant}\). The constant equals \(P_{\text{min}} + B_{\text{max}}^2/8\pi = (B_{\text{max}}^2/8\pi) (1 + \beta_{\text{min}})\) where \(B_{\text{max}} = \text{max}(B_{01}, B_{02})\), \(P_{\text{min}}\) is the minimum ion pressure, and \(\beta_{\text{min}} = P_{\text{min}}/(B_{\text{max}}^2/8\pi)\) is the minimum plasma beta. The choice of \(\beta_{\text{min}}\) specifies the initial pressure profile. Periodic boundary conditions are employed in both directions; the double tearing mode is set up by reflecting the profiles about \(z = 0\) to obtain \(B_z\), \(\rho\), and \(P_i\) for \(-L_z/2 < z < 0\).

[11] The standard implementation of the two-fluid equations [Shay et al., 2004] includes electron inertia by assuming that the ion velocity is negligible and the density is constant over the small length scales at which the inertia term is appreciable. We adopt the same convention, with the density given by \(\rho_0\). Improving this assumption requires extending the two-fluid formalism to capture the density gradient at electron scales or utilizing particle-in-cell simulations.

[12] We normalize magnetic fields and mass density to \(B_0\) and \(\rho_0\). Derived quantities are normalized using: velocities to the Alfvén speed \(c_{\text{A0}} = B_0/(4\pi\rho_0)^{1/2}\), lengths to the ion inertial length \(d_{\text{i0}} = (m_i\rho_0^2/4\pi\rho_0 c^2)^{1/2}\) where \(m_i\) is the ion mass, times to the ion cyclotron time \(t_{\text{c0}} = d_{\text{i0}}/c = \Omega_{\text{i0}}^{-1}\), electric fields to \(E_0 = c\rho_0 B_0/c\), and pressures to \(P_0 = \rho_0 c_{\text{A0}}^2\).

[13] The computational domain is of size \(L_x \times L_z = 204.8 \times 102.4 d_{\text{i0}}\) with a cell size of \(0.05 \times 0.05 d_{\text{i0}}\) and \(w_0 = 2.0 d_{\text{i0}}\). The electron mass \(m_e\) is fixed at \(m_e/25\), so the resolution is ostensibly high enough to resolve the electron layer. Reconnection is initiated using a coherent magnetic perturbation \(B_1 = -(0.01 B_0 \sqrt{L_z}/2\pi) \frac{\dot{\mathbf{z}}}{\mathbf{z}} \times \nabla [\sin(2\pi x/L_z) \sin(2\pi z/L_z)]\). We use \(\beta_{\text{min}} = 4\), which is chosen for numerical purposes. We do not expect the results to change for smaller values, but it should be checked in future work. There is no viscosity or resistivity, but fourth order diffusion with coefficient \(\sim 6.25 \times 10^{-6} d_{\text{i0}}^3\) is used in all of the equations to damp noise at the grid scale, where \(c_{\text{max}}\) is the maximum Alfvén speed on either side of the current sheet. Initial random perturbations on the magnetic field of amplitude \(5 \times 10^{-5} B_0\) and on the velocity of amplitude \(0.08 c_{\text{A0}}\) break the symmetry so that secondary magnetic islands are ejected.

[14] We perform four types of simulations: a reference run (Sym) with both \(B\) and \(\rho\) symmetric, runs with asymmetric \(B\) and symmetric (uniform) \(\rho\) (AB), runs with asymmetric \(\rho\) and symmetric \(B\) (AN), and both \(B\) and \(\rho\) asymmetric (ABN). The simulation parameters are given in Table 1.

[15] The systems are evolved until a steady state is reached as evidenced by the reconnection rate \(\dot{E}\), ion and electron outflow speeds \(v_{\text{out}}\) and \(v_{\text{eout}}\) and other quantities being relatively constant. There are two independent ways to measure \(\dot{E}\). One is the time rate of change of magnetic flux between the X-line and the O-line. A second is the convective electric field \(E = v_{\text{Bz}} / c\) upstream of the dissipation region. Just as in asymmetric Sweet-Parker reconnection [Cassak and Shay, 2007], the X-line moves in the inflow direction when \(B\) is asymmetric, so the inflow speed must be measured in the reference frame of the moving X-line. Values obtained using these two methods agree within 1–10%.

[16] Measuring the outflow speeds is not as simple. In the symmetric case, Shay et al. [2004] developed a way to evaluate \(v_{\text{out}}\) and \(v_{\text{eout}}\). A cut through the neutral line in the outflow direction reveals that the electron outflow increases

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### Table 1. Summary of Simulations Performed

<table>
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<th>Run</th>
<th>(B_{01})</th>
<th>(B_{02})</th>
<th>(\rho_{01})</th>
<th>(\rho_{02})</th>
<th>(P_1)</th>
<th>(P_2)</th>
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"Table 1. Summary of Simulations Performed"
to a peak value on the order of the electron Alfvén speed, then decreases before reaching a steady value. The ion outflow increases from zero at the X-line. Then, $v_{\text{out}}$ is defined as the peak in the electron outflow and $v_{\text{out}}$ as the velocity at which the ion outflow first exceeds the electron outflow.

[17] Generalizing this procedure to the asymmetric case is complicated because the X-line and the stagnation point are not colocated during asymmetric reconnection [Cassak and Shay, 2007]. We find the same is true in Hall reconnection provided $B$ is asymmetric, and in addition, the ions and electrons have different stagnation points. To apply the method of Shay et al. [2004], we find the location of the maximum in the ion outflow, which is not typically on the neutral line. Then, we take a cut from the electron stagnation point to the location of the maximum ion outflow. We apply the above procedure in this cut, defining $v_{\text{out}}$ as the maximum along that cut and $v_{\text{out}}$ as the point at which the ion outflow velocity exceeds the electron outflow velocity.

[18] For comparison, we have also used $v_{\text{out}}$ measured simply as the maximum ion outflow velocity. While the magnitudes of the velocities measured these two different ways differ by about 60%, we find that they scale with fields and densities the same way as the more complicated measurement, meaning that either method is acceptable to test the scaling.

[19] The predicted and measured values of $E$, $v_{\text{out}}$ and $v_{\text{out}}$, are shown in Figures 1, 2, and 3, respectively. On the vertical axes the simulation values are normalized to the value obtained in the symmetric case: $E_{\text{sym}} \simeq 0.0587 E_0$, $v_{\text{out}}_{\text{sym}} \simeq 0.685 c_{\alpha 0}$ and $v_{\text{out}}_{\text{sym}} \simeq 2.067 c_{\alpha 0}$. Since the field upstream of the electron layer is $B_e \sim 0.325$, the electron outflow speed in the symmetric run is comparable to the electron Alfvén speed $1.625 c_{\alpha 0}$ based on $B_c$, as expected [Shay et al., 2001]. On the horizontal axes are the predicted values from equations (1) and (2) using the asymptotic values for $B$ and $\rho$ and assuming that $\delta/L$ is independent of $B$ and $\rho$. For all the plots, the filled circles correspond to asymmetric field (AB) runs, the boxes to asymmetric density (AN), and the asterisks to both density and field asymmetric (ABN).

[20] The plots reveal very good agreement between the theory and simulations for all three quantities. The agreement for $E$ implies that $\delta/L$ is independent of (or only weakly dependent on) $B$ and $\rho$. Similar results are found using asymmetric field (AB) simulations in a system twice as large (not shown). We have confirmed that increasing the fourth order diffusion coefficient does not adversely affect the results.

4. Observational Signatures

[21] We briefly comment on some signatures of asymmetric Hall reconnection; a more complete study is forthcoming. Consistent with previous simulations, we observe that the island preferentially ‘bulges’ toward the weak field side. The effect does not occur when $B$ is symmetric, even if the densities differ, also consistent with previous simulations. The bulge occurs because it is easier for the newly reconnected field lines to bend the weaker field than the stronger field upstream of the island [Cassak and Shay, 2007].

[22] As mentioned in the previous section, the current sheet moves in the direction of the inflow toward the plasma.
with the stronger magnetic field in the reference frame of the simulation, consistent with previous simulations [Ugai, 2000; Jin et al., 2000; Lin, 2001; Cassak and Shay, 2007]. The drift occurs to a much lesser extent for asymmetric B, even with asymmetric $\rho$. This drift occurs because it takes more energy to bend the magnetic fields on the strong field side than it takes for the X-line to propagate towards the strong field [Ugai, 2000; Cassak and Shay, 2007]. It will be difficult to discern this effect observationally because the breathing motion of the magnetopause is faster than the X-line speed. Indeed, recent observations by Mozer et al. [2008] found the magnetopause moving toward the magnetosheath (rather than the magnetosphere) at a speed of 14 km/sec, while the inflow speed on the magnetosphere side was $cE/B \sim 3.6$ km/sec.

[23] It was predicted that a generic feature of asymmetric reconnection is that the X-line and stagnation point are not colocated, implying a bulk flow through the X-line [Cassak and Shay, 2007]. This has been observed in global magnetosphere simulations [Siscoe et al., 2002; Dorelli et al., 2004] and reconnection simulations using MHD with an anomalous resistivity [Birn et al., 2008] and particle-in-cell [Pritchett, 2008] codes. We find that, for the cases with asymmetric fields (AB and ABN), the X-line is not colocated with the stagnation point and the ion and electron stagnation points are distinct as well. This result is potentially useful for explaining the puzzling particle trajectories inferred from recent satellite observations [Mozer et al., 2008].

5. Applications and Discussion

[24] In this paper, we verified that scaling laws derived by Cassak and Shay [2007] for the reconnection rate and ion outflow speed during asymmetric reconnection apply to Hall reconnection. The electron outflow speed obeys the same scaling relation as the ion outflow speed. We found that the normalized reconnection rate $E$ is independent of the asymmetry in the density and field strength, maintaining a value on the order of 0.1. This adds credence to the claim that $E$ is independent of external parameters in Hall reconnection [Shay et al., 1999; Huba and Rudakov, 2004; Shay et al., 2004]. It was previously shown that $E' \sim 0.1$ for asymmetric fast reconnection [Birn et al., 2008; Borovsky et al., 2008], but the present result confirms the scaling in a self-consistent collisionless system, as is relevant for the magnetosphere.

[25] The present result has been applied to solar wind-magnetospheric coupling by Borovsky [2008], who assumed the reconnection rate at the magnetopause is given by equation (1) based on local quantities as opposed to the solar wind electric field. The local parameters were related to solar wind parameters through pressure balance arguments. The correlation between solar wind data and geomagnetic indices was as good as the best previous model, which used fitting techniques [see also Turner et al., 2008]. The model assumed that $\beta/L \sim 0.1$, and the present result justifies this assumption for collisionless plasmas in the magnetosphere. Whether the present results apply to a three-dimensional system such as the magnetopause has been called into question [Dorelli, 2008]; future work is required.

[26] There are a few notable limitations of the present simulations. As mentioned earlier, the density asymmetry is not properly accounted for in the electron inertia term in the two-fluid model utilized here and the simulations used $\beta_{\text{min}} = 4$, much larger than in the magnetosphere. Also, Swisdak et al. [2003] showed $E$ is reduced when there is a density gradient across the dissipation region in the presence of a guide field because of diamagnetic drifts in the outflow direction, so the extension of these results to include a guide field is imperative.

[27] Another limitation is that resolving the electron layer presents a strong constraint on the degree of asymmetry which can reliably be simulated. Cassak and Shay [2007] showed that the position of the X-line and stagnation point are offset from the center of the current sheet, with larger offsets for larger asymmetries. One must resolve the electron current layer with more cells $n_d$ across the dissipation region that the degree of asymmetry in the densities $\rho_{O2}/\rho_{O3}$ or fields $B_{O2}/B_{O1}$, i.e., one must have

$$n_d > B_{O2}/B_{O1} \times \rho_{O2}/\rho_{O3}$$

in order to make reliable predictions of asymmetric Hall reconnection. The present simulations are limited to symmetries of less than four. Simulations with a larger asymmetry in the magnetic field result in a reconnection rate which continues to obey equation (1), but in which the electron layer becomes distorted and the scaling of the velocity and other quantities break down. A question for future work is what the structure of the dissipation region.

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References


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