**Kinetic equilibrium solution to the Vlasov equation in cylindrical geometry**

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**Motivation**
- Solve Vlasov-Poisson equation in cylindrical geometry for a pressure gradient comparable in length to an ion gyroradius in the NRL Space Chamber
- Estimate the self-consistent electric field perp and parallel to the magnetic field and the resulting temperature anisotropy in the gradient region
- Explain the emergence of observed broadband wave emission using the equilibrium solution

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**NRL Space Chamber Experimental Conditions**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>n (cm⁻³)</td>
<td>10⁻¹⁰⁻¹⁰⁻⁶</td>
</tr>
<tr>
<td>T_e (eV)</td>
<td>0.1-1.4</td>
</tr>
<tr>
<td>B (G)</td>
<td>Up to 250</td>
</tr>
<tr>
<td>μ_e (mm)</td>
<td>0.1-0.7</td>
</tr>
<tr>
<td>f_p (Hz)</td>
<td>10⁻⁵⁻¹⁰⁻³</td>
</tr>
<tr>
<td>f_e (Hz)</td>
<td>10⁻¹⁰⁻⁴⁻¹⁰⁻³ (Ar²)</td>
</tr>
<tr>
<td>f_h (Hz)</td>
<td>10⁻¹⁻¹⁰⁻³</td>
</tr>
</tbody>
</table>

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**Vlasov Solution Constructed from Constants of Motion**

\[ f_0(H, r_g) = \frac{N_e}{(2\pi)^{3/2}|\Psi_0|} \exp \left( -\frac{H^2}{2|\Psi_0|^2} \right) \]

\[ Q_0(r_g) = \left( \frac{r_g - R_i}{R_o - R_i} \right), \quad \lambda_i^2 < r_g^2 < \lambda_e^2 \]

**Equilibrium Solution Replicates Space Chamber Conditions on \( \rho_1 \) Scale**

Model density profiles are obtained as the zeroth moment of \( f_0 \).

**Temperature Anisotropy Creates Instabilities (What Instabilities?)**

- Higher moments of \( f_0 \) are computed to give the velocity and temperature profiles across the gradient region.

**Broadband Waves Observed in Space Chamber**

- Waves exhibit burning, chirping, and amplitude substructure
- Burst frequency falls into two frequency bands above and below electron cyclotron frequency
- High frequency band appears when low bias voltage is applied to the interior plasma column
- High-frequency waves exist where streaming electrons and pressure gradients coincide

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**Conclusions**
- Model reproduces density and electric field profiles in NRLSPC
- Ion and electron layer separation produces current across the density gradient
- Temperature anisotropy arises do to \( E_L \) which can create instabilities

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**Future Work**

- Investigate collisional effects on the equilibrium
- Consider piecewise linear \( Q(r) \) to eliminate kinks at the layer boundary
- Develop solution for an expanding magnetic field

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This work was performed at NRL in the summer of 2018, and would not be possible without essential contributions from Chris Crabtree, Guru Ganguli, Erik Tejero, and Lon Enloe.

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**LIFE Waveforms**

- A 15 V/m electric field nearly doubles the orbital frequency of 0.5 eV Ar ions in a 100 G magnetic field.
- The orbital frequency of 2 eV electrons is unaffected by a 1000 V/m electric field in a 100 G magnetic field.

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**Applicability to Helix-Leia System**

The analysis thus far was performed assuming a uniform magnetic field. A magnetic field gradient in the direction of the field will cause a change on the local plasma parameters along the field lines, giving rise to a changing potential \( \Phi(r) \), and thus a parallel electric field, \( E_p = \partial \Phi(r)/\partial z \). This parallel electric field may accelerate particles along the field creating inhomogeneous beams or flows, as seen in the Helix-Leia system at WVU.

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**References**

- Crabtree, Guru Ganguli, Erik Tejero, and Lon Enloe.
- A new magnetic field geometry is reflected in the Hamiltonian.