Main Ideas Today: Density, Pressure & Buoyancy
Why does ice float?

BECAUSE IT'S COLD, ICE WANTS TO GET WARM, SO IT GOES TO THE TOP OF LIQUIDS IN ORDER TO BE NEARER TO THE SUN.

IS THAT TRUE? LOOK IT UP AND FIND OUT.

I SHOULD JUST LOOK STUFF UP IN THE FIRST PLACE. YOU CAN LEARN A LOT, TALKING TO ME.
Density is \[ \rho = \frac{M}{V} = \frac{\text{mass}}{\text{volume}} \]

The values of density for a substance vary slightly with temperature since volume is temperature dependent.

The various densities indicate the average molecular spacing in a gas is much greater than that in a solid or liquid.

<table>
<thead>
<tr>
<th>Substance</th>
<th>( \rho (\text{kg/m}^3) )</th>
<th>Substance</th>
<th>( \rho (\text{kg/m}^3) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>1.29</td>
<td>Ice</td>
<td>0.917 \times 10^3</td>
</tr>
<tr>
<td>Aluminum</td>
<td>( 2.70 \times 10^3 )</td>
<td>Iron</td>
<td>7.86 \times 10^3</td>
</tr>
<tr>
<td>Benzene</td>
<td>( 0.879 \times 10^3 )</td>
<td>Lead</td>
<td>11.3 \times 10^3</td>
</tr>
<tr>
<td>Copper</td>
<td>( 8.92 \times 10^3 )</td>
<td>Mercury</td>
<td>13.6 \times 10^3</td>
</tr>
<tr>
<td>Ethyl alcohol</td>
<td>( 0.806 \times 10^3 )</td>
<td>Oak</td>
<td>0.710 \times 10^3</td>
</tr>
<tr>
<td>Fresh water</td>
<td>( 1.00 \times 10^3 )</td>
<td>Oxygen gas</td>
<td>1.43</td>
</tr>
<tr>
<td>Glycerin</td>
<td>( 1.26 \times 10^3 )</td>
<td>Pine</td>
<td>0.373 \times 10^3</td>
</tr>
<tr>
<td>Gold</td>
<td>( 19.3 \times 10^3 )</td>
<td>Platinum</td>
<td>21.4 \times 10^3</td>
</tr>
<tr>
<td>Helium gas</td>
<td>( 1.79 \times 10^{-1} )</td>
<td>Seawater</td>
<td>1.03 \times 10^3</td>
</tr>
<tr>
<td>Hydrogen gas</td>
<td>( 8.99 \times 10^{-2} )</td>
<td>Silver</td>
<td>10.5 \times 10^3</td>
</tr>
</tbody>
</table>
The sphere on the right has twice the mass and twice the radius of the sphere on the left.

Compared to the sphere on the left, the larger sphere on the right has

A. twice the density.
B. the same density.
C. 1/2 the density.
D. 1/4 the density.
E. 1/8 the density.

\[ V_{sphere} \propto R^3 \]

\[ \rho = \frac{M}{V} = \frac{mass}{volume} \]

Not necessarily made of same material.
When a dense object (or any object) pushes against you, it applies pressure (or stress).

Pressure = Force / Area

Unit of pressure is pascal (Pa)  
$1 \text{ Pa} = 1 \text{ N/m}^2$

Pressure depends on the area over which the force is spread  
(Also known as stress in solid materials.)
Pressure always pushes perpendicular to the surface.

A man sits on a four-legged chair with his feet off the floor. The combined mass of the man and chair is 95 kg. If the chair legs are circular and have a radius of 0.50 cm at the bottom, what pressure does each leg exert on the floor?

What could you do to reduce pressure and chance of scratching?
Pressure & Depth

- Assume the density is the same throughout the fluid.
- Fluids have pressure that varies with depth.
- If a fluid is at rest in a container, all portions of the fluid must be in static equilibrium.
- All points at the same depth must be at the same pressure.
- Otherwise, the fluid would move (not equilibrium).

\[ \sum \vec{F} = 0 \]

Reminder: \( P = \frac{\text{Force}}{\text{Area}} \)

Examine the darker region, a sample of liquid within a cylinder.
- It has a cross-sectional area \( A \).
- Extends from depth \( d \) to \( d + h \) below the surface.

Here density is not the same (why it separates).
Pressure & Depth

- Assume the density is the same throughout the fluid.

- Fluids have pressure that varies with depth.

- If a fluid is at rest in a container, all portions of the fluid must be in static equilibrium:
  \[ \sum \vec{F} = 0 \]

- All points at the same depth must be at the same pressure.
  Otherwise, the fluid would move (not equilibrium).

Examine the darker region, a sample of liquid within a cylinder:

- It has a cross-sectional area \( A \).
- Extends from depth \( d \) to \( d + h \) below the surface.

What forces act on this box of liquid?

Area \( A \) coming out at you (hidden)

Reminder: \( P = \text{Force} / \text{Area} \)

Warning: Derivation coming.
Pressure & Depth

\[ P = \frac{F}{A} \text{ or } F = PA \]

- The three forces are:  
  - Downward force on the top, \( P_0A \)
  - Upward on the bottom, \( PA \)
  - Gravity acting downward, \( Mg \)

- The mass can be found from the density: \( M = \rho V = \rho Ah \)

- Since the net force must be zero:  
  - This chooses upward as positive

- Solving for the pressure gives

  \[ P = P_0 + \rho gh \]

- The pressure \( P \) at a depth \( h \) below a point in the liquid at which the pressure is \( P_0 \) is greater by an amount \( \rho gh \)
Calculate the absolute pressure at the bottom of a freshwater lake at a depth of 27.5 m. Assume the density of the water is 1000 kg/m³ and the air above is at a pressure of 101.3 kPa.
Good Ideas Come To You When Relaxed

• [https://www.youtube.com/watch?v=ijj58xD5fDI](https://www.youtube.com/watch?v=ijj58xD5fDI)
Archimedes’ Principle

If the weight of the displaced fluid equals the weight of the object, the object floats.

\[ F_b = F_g \]

Floating Object Only

**Buoyant Force**

- The **buoyant force** is the upward force exerted by a fluid on any immersed object.
- The parcel is in equilibrium.

\[ F_b = W_{\text{FluidDisplaced}} \]

Any object **completely or partially** submerged in a fluid is buoyed up by a force equal to the weight of the fluid displaced.
Two blocks (A and B) have the same size and shape. **Block A floats** in the water, but **Block B sinks** in the same **water**. Which block has the larger buoyant force on it?

\[ F_b = W_{\text{FluidDisplaced}} \]

A. Block A has the larger buoyant force on it.
B. Block B has the larger buoyant force on it.
C. Neither; they have the same.
D. Not enough information
Archimedes’s Principle

The magnitude of the buoyant force always equals the weight of the fluid displaced by the object

$$F_b = M_{\text{Water Displaced}} g = \rho_{\text{fluid}} g V$$

Archimedes’s Principle does not refer to the makeup of the object experiencing the buoyant force

Two ways to find $F_B$

As stated before:

1. The pressure at the top of the cube causes a downward force of $P_t A$
2. The pressure at the bottom of the cube causes an upward force of $P_b A$

$$F_b = (P_b - P_t) A = \rho_{\text{fluid}} V \cdot g$$
If the weight of the displaced water is less than the weight of the object, the object sinks.

Therefore, if the average density of the object is more than the density of water, it sinks.

Object sinks

- The upward buoyant force is \( F_b = \rho_{\text{fluid}} g V = \rho_{\text{fluid}} g V_{\text{object}} \)
- The downward gravitational force is \( F_g = \rho_{\text{object}} g V_{\text{object}} \)
- The net force is \( F_b - F_g = (\rho_{\text{fluid}} - \rho_{\text{object}}) g V_{\text{object}} \)
How can a steel ship float?

The hull contains mostly air and displaces a lot of water...enough so that $F_b = F_g$ and it floats.
Summary for a Floating Object

- Object in equilibrium
- Buoyant force is balanced by force of gravity
- Volume of the fluid displaced corresponds to volume of the object beneath the fluid level

\[ F_b = F_g \]
\[ \rho_{fluid} g V_{fluid} = \rho_{object} g V_{object} \]
\[ \frac{V_{fluid}}{V_{object}} = \frac{\rho_{object}}{\rho_{fluid}} \]
Archimedes’s Principle, Iceberg Example

- What fraction of the iceberg is below water?
- The iceberg is only partially submerged and so $\frac{V_{\text{fluid}}}{V_{\text{object}}} = \frac{\rho_{\text{object}}}{\rho_{\text{fluid}}}$ applies
- The fraction below the water will be the ratio of the volumes ($\frac{V_{\text{water}}}{V_{\text{ice}}}$)

$V_{\text{ice}}$ is the total volume of the iceberg

$V_{\text{water}}$ is the volume of the water displaced
  - This will be equal to the volume of the iceberg submerged

About 89% of the ice is below the water’s surface
The average human has a density of 945 kg/m$^3$ after inhaling and 1020 kg/m$^3$ after exhaling. Without making any swimming movements, what percentage of the human body would be above the surface in the Dead Sea (a lake with a water density of about 1230 kg/m$^3$) after inhaling and after exhaling?

\[
\frac{\rho_{\text{object}}}{\rho_{\text{fluid}}} = \frac{V_{\text{fluid}}}{V_{\text{object}}}
\]
You hold a 0.54 kg rubber ball with a diameter of 25 cm just below the water’s surface in your swimming pool. With what force do you have to apply to keep the ball from popping back up above the water?

Density of freshwater = 1000 kg/m³
Pressure Measurements:

Manometer

- A device for measuring the pressure of a gas contained in a vessel.
- One end of the U-shaped tube is open to the atmosphere.
- The other end is connected to the pressure to be measured.
- Pressure at B is $P_0 + \rho gh$.

Absolute vs. Gauge Pressure

- $P = P_0 + \rho gh$
- $P$ is the absolute pressure.
- The gauge pressure is $P - P_0$.
  - This is also $\rho gh$.
  - This is what you measure in your tires.

Not on test.
Chapter/Section: Clicker #答=Answer
Ch 9: 5=D, 6=B
\[ p = \frac{m}{V} \left( \frac{2m}{(\frac{4}{3})\pi(2r)^3} \right) \frac{2}{8} \frac{1}{4} \]

\[ p = \frac{F}{A} = \frac{mg}{4} \]
\[ m = 9.5 \text{ kg} \]
\[ r = 0.50 \text{ cm} \]

\[ = \frac{(9.5 \text{ kg})(9.8 \text{ m/s}^2)}{(\pi (0.50 \text{ cm} \div 100 \text{ cm}))^2} = 3.0 \times 10^6 \text{ Pa} \]

\[ p = p_0 + \rho gh \]
\[ h = 27.5 \text{ m} \]
\[ p = 1000 \text{ kg/m}^3 \]
\[ p_0 = 101.3 \text{ kPa} \]

\[ p = 101.3 \text{ kPa} + (1000 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(27.5 \text{ m}) \]
\[ = 101.3 \text{ kPa} + 269.5 \text{ kPa} \]
\[ = 370.8 \text{ kPa} \]

\[ \% \text{ submerged} = \frac{V_{\text{fluid}}}{V_{\text{object}}} = \frac{P_{\text{object}}}{\rho_{\text{fluid}}} = \frac{0.917 \times 10^3 \text{ Pa}}{1.025 \times 10^3 \text{ Pa}} = 0.89 \]

\[ \% \text{ inhale} \]
\[ \frac{p_{\text{inhale}}}{\rho_{\text{water}}} = \frac{945 \text{ kg/m}^3}{1230 \text{ kg/m}^3} = 0.77 \]

\[ \% \text{ exhale} \]
\[ \frac{1020 \text{ kg/m}^3}{1230 \text{ kg/m}^3} = 0.83 \]
\[ m = 0.54 \text{ kg} \quad d = 25 \text{ cm} \]
\[ r = \frac{25 \text{ cm}}{2} \left( \frac{m}{\text{kg} \cdot \text{m}} \right) = 0.125 \text{ m} \]
\[ \rho_{\text{water}} = 1000 \text{ kg/m}^3 \]

\[ F_b = \rho_{\text{fluid}} V_{\text{obj}} g \quad F_p = ? \]

\[ \sum F = 0 \]
\[ F_b - F_g - F_p = 0 \]
\[ \rho_{\text{fluid}} V_{\text{obj}} g - mg = F_p \]

\[ F_p = \left( 1000 \text{ kg/m}^3 \right) \left( \frac{4}{3} \pi \left( 0.125 \text{ m} \right)^3 \right) (9.8 \text{ m/s}^2) - (0.54 \text{ kg})(9.8 \text{ m/s}^2) \]
\[ = 75 \text{ N} \]