Effect of ion cyclotron parametric turbulence on the generation of edge suprathermal ions during ion cyclotron plasma heating

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The effect of parametric ion cyclotron turbulence on the heating of cold and suprathermal ions in the edge of a tokamak plasma during injection of high radio frequency (rf) power in the ion cyclotron resonance frequency range is studied. The maximum turbulent heating rates for cold edge ions and suprathermal edge ions are calculated analytically for ion cyclotron turbulence driven by rf heating at the plasma edge. It is demonstrated that the maximum turbulent ion-heating rate for suprathermal ions is insufficient to explain the observed heating of edge ions. Therefore, the excitation of ion cyclotron turbulence by rf heating systems in the plasma edge is unlikely to be responsible for the experimentally observed large population of suprathermal ions in the edge of tokamak plasmas.
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I. INTRODUCTION

The generation of suprathermal ions is regularly observed in the edge and scrape off layer of tokamak plasmas during injection of high power radio frequency (rf) waves in the ion cyclotron resonance frequency (ICRF) range. For example, fast ion production in the scrape off layer was noted during ICRF fast wave heating on the ASDEX (ion energies of 1–2 keV)[1] and on the Alcator C-Mod (ion energies greater than 5 keV)[2] tokamaks. On the Japan Tokamak-60 (JT-60), charge-exchange neutral particle analysis indicated that ions with energy of about 5 keV were created near the plasma edge during ICRF.[3] Charge-exchange measurements also indicated strong heating of ions in the plasma edge (the hydrogen and deuterium tail temperatures were 5.5 and 1.8 keV, respectively) during high power ion Bernstein wave injection in the Doublet III-D (DIII-D) tokamak.[4]

The primary objective of those high power rf injection experiments was bulk plasma heating. rf power absorption in the plasma edge reduces the power available for central heating and is undesirable. Poorly confined high energy ions in the plasma edge contribute to impurity generation and also affect the stability of enhanced confinement operational regimes. Specifically, edge ion heating is observed in high confinement discharges (H-mode discharges) without edge-localized modes.[2] Therefore, edge ion heating may play an important role in tokamak performance.

The experimental data are consistent with production of suprathermal ions in the plasma edge due to the interaction of the ICRF with the boundary plasma. A high-energy tail appears in the measured perpendicular ion velocity space distribution and no significant parallel ion heating is observed. Formation of the population of the suprathermal ions is observed only when a threshold power level is exceeded and the magnitude of the threshold power depends on the heating method chosen. In the Alcator C-Mod tokamak, suprathermal ions appeared in the plasma edge during hydrogen minority ion heating in deuterium plasmas only at rf power levels above approximately 500 kW. However, when the pump wave frequency is near an ion cyclotron harmonic in the scrape-off layer (SOL), the power threshold for the formation of a population of suprathermal ions is much smaller, below 10 kW. The formation of the hot tail in the perpendicular ion distribution and the wide variation in threshold rf powers suggest that the edge ion heating is caused by the rf fields near the antenna and that nonlinear processes are responsible for the edge ion heating.

The excitation of parametric instabilities is another nonlinear process that has been observed in the scrap-off layer of several ICRF-heated tokamaks such as ASDEX,[5] Joint European Torus (JET),[6] Japan Tokamak-60 (JT-60),[3] Alcator C-Mod,[2] and DIII-D[4] when a rf power threshold was exceeded. The instabilities were localized to the region near the rf antenna in the cold edge plasma. Typical edge plasma parameters included a plasma density of $n \sim 10^{13}\ \mathrm{cm}^{-3}$, ion temperature of $T_i \geqslant 3\ \mathrm{eV}$, and electron temperature of $T_e = 10-20\ \mathrm{eV}$ in the Alcator C-Mod tokamak[2] and in the DIII-D tokamak, $n \sim 2\times 10^{11}\ \mathrm{cm}^{-3}$ and $T_e \sim T_i \sim 10\ \mathrm{eV}$.[4] Because the observed rf power thresholds for formation of high energy ion tails were comparable with the rf power thresholds for the parametrically driven, quasimode decay instabilities, it was suggested that generation of suprathermal ions in the cold edge plasma is due to ion cyclotron damping of the ion cyclotron quasimode.[1–4] However, to the best of our knowledge, no conclusive analytical or experimental studies on the role of parametric decay instabilities or parametric turbulence in the formation of suprathermal ions have been performed. In fact, the theoretical justification for concluding that the generation of suprathermal ions in the cold edge plasma is due to ion cyclotron damping of the ion cyclotron quasimode[1–4] is based on calculations of growth rates and power thresholds of possible parametric decay instabilities for the case of small displacements of ions with respect to electrons in the pumping wave.[7,8] It is true that for waves with wavelengths much longer than the typical electrostatic probe size used in the edge plasmas of tokamaks, only small ion displacements in the wave field need to be considered theoretically. However, a complete theoretical analysis should include the possibility that short wavelength modes can also be unstable and give rise to ion heating.

Previous theoretical calculations suggested that for the same parameters at which parametric decay into an ion Bernstein wave and an ion cyclotron quasimode are observed in experiment,[1–6] the short wavelength electrostatic ion cyclotron kinetic parametric instability[9] and kinetic parametric ion cyclotron turbulence[10] may be also be excited. The short scale nature of these nonlinearly driven processes makes them undetectable by typical size electrostatic probes. The ion cyclotron kinetic parametric instability has a maximum growth rate for ion cyclotron waves with wavelengths comparable to the relative ion-electron displacements in the pumping field. For intense rf pumping fields, the growth rate of the ion cyclotron kinetic parametric instability is significantly larger than that of the quasimode decay instability. Therefore, the growth of the ion cyclotron kinetic parametric instability will control the level of ion cyclotron turbulence and the ion heating rate. Under such conditions, the ion heating rate due to the ion cyclotron kinetic parametric instability may be taken as an upper bound on the anomalous heating rate due to the interaction of ions with ion cyclotron parametric turbulence. In this paper, we present linear and nonlinear analysis of parametrically excited ion cyclotron turbulence and estimate its role in the formation of a population of suprathermal ions. In Sec. II the basic equations are presented. The linear and nonlinear theories of ion cyclotron parametric turbulence, as well as an analysis of the turbulent heating of electrons and ions in the cold edge plasma are presented in Sec. III. The role of the ion cyclotron parametric turbulence in the generation of the suprathermal ions is discussed in Sec. IV and the work is summarized in Sec. V.

II. BASIC EQUATIONS

We begin our analysis of the linear stage of electrostatic ion-cyclotron instabilities with a Fourier transformation of the electrostatic potential, performed in the oscillating system of coordinates, $\varphi_i(\mathbf{k},\omega)$, moving with the ion velocity in the confining magnetic field $\mathbf{B}$ and rf pumping electric field $\mathbf{E}_0(\mathbf{r},t)=\mathbf{E}_0\cos(\omega_0 t-\mathbf{k}_{0\parallel}\mathbf{r})$. The electric field has frequency ω_0 and wave number $\mathbf{k}_{0\parallel}\|\mathbf{B}$.[7–10] We assume $\mathbf{k}_{0\perp}=0$ ($\mathbf{k}_{0\perp}\perp\mathbf{B}$) because in the case of electromagnetic fast wave heating experiments, the parametrically excited waves are electrostatic modes with very short wavelengths across the magnetic field compared to the pump wave. In the case of IBW wave excitation experiments, $k_{0\perp}\rho_i\ll 1$ (where $\rho_i=v_{\mathrm{Ti}}/\omega_{\mathrm{ci}}$ is a thermal Larmor radius, v_{Ti} is the ion thermal velocity, and ω_{ci} is the cyclotron frequency of ions), in the field of the pump wave, but $k_\perp\rho_i\gg 1$ for the much shorter wavelength parametrically excited waves. Nevertheless, finite $\mathbf{k}_{0\parallel}$ may, in some cases be important in maximizing the growth rate of the instability. Under these assumptions, the equation governing the linear evolution of the Fourier transformed electrostatic potential is given by[10]

$$\varepsilon(\mathbf{k},\omega)\varphi_i(\mathbf{k},\omega)+\sum_{m\neq 0}\sum_{n=-\infty}^{\infty}J_n(a_{ie})J_{n+m}(a_{ie})e^{im\delta_{ie}}\delta\varepsilon_e(\mathbf{k}-n\mathbf{k}_{\parallel 0},\omega-n\omega_0)\times\varphi_i(\mathbf{k}+m\mathbf{k}_{0\parallel},\omega+m\omega_0)=0, \tag{1}$$

$$\varepsilon(\mathbf{k},\omega)=1+\delta\varepsilon_i(\mathbf{k},\omega)+\sum_{\nu=-\infty}^{\infty}J_\nu^2(a_{ie})\,\delta\varepsilon_e(\mathbf{k}-\nu\mathbf{k}_{\parallel 0},\omega-\nu\omega_0) \tag{2}$$

and $\delta\varepsilon_{i,e}(\mathbf{k},\omega)$ is the ion (electron) dielectric permittivity

$$\delta\varepsilon_\alpha(\mathbf{k},\omega)=\frac{1}{k^2\lambda_{D\alpha}^2}\left[1+i\sqrt{\pi}z_{0\alpha}\sum_{n=-\infty}^{\infty}W(z_{n\alpha})\times e^{-k_\perp^2\rho_{L\alpha}^2}I_n(k_\perp^2\rho_\alpha^2)\right]\quad(\alpha=i,e), \tag{3}$$

$\lambda_{D\alpha}$ is the Debye radius, $\rho_\alpha=v_{T\alpha}/\omega_{c\alpha}$ is the thermal Larmor radius, $v_{T\alpha}$ is the thermal velocity, $\omega_{c\alpha}$ is the cyclotron frequency of particles species α:

$$W(z)=\exp(-z^2)\left(\operatorname{sgn}k_\parallel+\frac{2i}{\sqrt{\pi}}\int_0^z e^{t^2}dt\right),$$

$$z_{n\alpha}=\frac{\omega-n|\omega_{c\alpha}|}{\sqrt{2}k_\parallel v_{T\alpha}},\quad\text{and}\quad z_{0\alpha}=\frac{\omega}{\sqrt{2}k_\parallel v_{T\alpha}}.$$

The parameters a_{ie} and δ_{ie} are defined in Refs. 7–10 and are equal to

$$a_{ie}=\left\{\left[\sum_{\alpha=i,e}\frac{|e_\alpha|}{m_\alpha}\left(\frac{\mathbf{E}_{0\perp}\cdot\mathbf{k}_\perp}{\omega_0^2-\omega_{c\alpha}^2}+\frac{E_\parallel k_\parallel}{\omega_0^2}\right)\right]^2+\left[\sum_{\alpha=i,e}\frac{|e_\alpha|}{m_\alpha}\frac{\omega_{c\alpha}}{\omega_0}\frac{\mathbf{B}_0\cdot(\mathbf{k}_\perp\times\mathbf{E}_{0\perp})}{(\omega_0^2-\omega_{c\alpha}^2)^2B_0}\right]^2\right\}^{1/2},$$

$$\delta_{ie}=\left[\sum_{\alpha=i,e}\frac{|e_\alpha|}{m_\alpha}\left(\frac{\mathbf{E}_{0\perp}\cdot\mathbf{k}_\perp}{\omega_0^2-\omega_{c\alpha}^2}+\frac{E_\parallel k_\parallel}{\omega_0^2}\right)\right]\times\left[\sum_{\alpha=i,e}\frac{|e_\alpha|}{m_\alpha}\frac{\omega_{c\alpha}}{\omega_0}\frac{\mathbf{B}_0\cdot(\mathbf{k}_\perp\times\mathbf{E}_{0\perp})}{(\omega_0^2-\omega_{c\alpha}^2)^2B_0}\right]^{-1}.$$

Equation (1) is a difference equation for φ_i and in general must be solved numerically. An analytical solution is possible when, under certain conditions, it is possible to discard all but one or two terms in summation over m. These conditions are: (1) small relative displacement of ions with respect to electrons (or ions of another species) ξ_{ie} (ξ_{ii}) in the pumping field with respect to the wavelength $\lambda=2\pi/k$ of unstable oscillation, i.e., $a_{ie}\sim k\xi_{ie}\ll 1$[7,8] or (2) a small value of the ratio of electron to ion dielectric permittivities, $|\delta\varepsilon_e/\delta\varepsilon_i|<1$.[9,10] In these cases, the following procedure is useful and yields order of magnitude estimates of each term so that only the largest terms are retained in the sum: rewriting Eq. (1):

$$\varphi_i(\mathbf{k}+t\mathbf{k}_{\|0},\omega+t\omega_0)=-\frac{1}{\varepsilon(\mathbf{k}+t\mathbf{k}_{\|0},\omega+t\omega_0)}\sum_{m\neq 0}\sum_{n=-\infty}^{\infty}J_n[a_{ie}(\mathbf{k})]J_{n+m}[a_{ie}(\mathbf{k})]\times e^{im\delta_{ie}(\mathbf{k})}\cdot\delta\varepsilon_e[\mathbf{k}-(n-t)\mathbf{k}_{\|0},\omega-(n-t)\omega_0]\varphi_i[\mathbf{k}+(m+t)\mathbf{k}_{\|0},\omega+(m+t)\omega_0], \quad (4)$$

where t is an integer. The substitution of Eq. (4) into Eq. (1) to replace the $\varphi_i(\mathbf{k}+t\mathbf{k}_{\|0},\omega+t\omega_0)$ term yields

$$\Bigg\{\varepsilon(\mathbf{k},\omega)-\sum_{m\neq 0}\sum_{n=-\infty}^{\infty}\sum_{s=-\infty}^{\infty}J_n(a_{ie})J_{n+m}(a_{ie})J_s(a_{ie})J_{s-m}(a_{ie})\times\frac{\delta\varepsilon_e(\mathbf{k}-n\mathbf{k}_{\|0},\omega-n\omega_0)\,\delta\varepsilon_e[\mathbf{k}-(s-m)\mathbf{k}_{\|0},\omega-(s-m)\omega_0]}{\varepsilon(\mathbf{k}+m\mathbf{k}_{\|0},\omega+m\omega_0)}\Bigg\}\varphi_i(\mathbf{k},\omega)$$
$$-\sum_{m\neq 0}\sum_{r\neq 0}\sum_{n=-\infty}^{\infty}\sum_{s=-\infty}^{\infty}J_n(a_{ie})J_{n+m}(a_{ie})J_{s+r}(a_{ie})J_s(a_{ie})e^{i(r+m)\delta_{ie}(\mathbf{k})}\times\frac{\delta\varepsilon_e(\mathbf{k}-n\mathbf{k}_{\|0},\omega-n\omega_0)\,\delta\varepsilon_e[\mathbf{k}-(s-m)\mathbf{k}_{\|0},\omega-(s-m)\omega_0]}{\varepsilon(\mathbf{k}+m\mathbf{k}_{\|0},\omega+m\omega_0)}\times\varphi_i[\mathbf{k}+(r+m)\mathbf{k}_{\|0},\omega+(r+m)\omega_0]=0. \quad (5)$$

The process is then repeated *ad infinitum* to obtain $\varphi_i(\mathbf{k},\omega)$. This type of calculation is similar to the procedure for obtaining the diagonal terms in the renormalization of the wave kinetic equation.[11] In the case when $|\delta\varepsilon_e/\delta\varepsilon_i|<1$, this procedure (at least asymptotically) converges. Essentially, terms of the order of $O(|\delta\varepsilon_e/\delta\varepsilon_i|^n)\ll 1$ are added at the nth step of the iteration. It is important to note that the product of multiple Bessel functions can be small even in cases where $|\delta\varepsilon_e/\delta\varepsilon_i|\sim 1$. Omitting terms of order $(|\delta\varepsilon_e|/|\delta\varepsilon_i|)^3\sim 1$, Eq. (5) becomes

$$\Bigg\{\varepsilon(\mathbf{k},\omega)-\sum_{m\neq 0}\sum_{n=-\infty}^{\infty}\sum_{s=-\infty}^{\infty}J_n(a_{ie})\,J_{n+m}(a_{ie})J_s(a_{ie})J_{s-m}(a_{ie})\times\frac{\delta\varepsilon_e(\mathbf{k}-n\mathbf{k}_{0\|}+m\mathbf{k}_{0\|},\omega-n\omega_0)\,\delta\varepsilon_e[\mathbf{k}-(s-m)\mathbf{k}_{0\|},\omega-(s-m)\omega_0]}{\varepsilon(\mathbf{k}+m\mathbf{k}_{0\|},\omega+m\omega_0)}\Bigg\}\varphi_i(\mathbf{k},\omega)=0. \quad (6)$$

The first term in Eq. (6) describes the development of kinetic parametric instabilities;[9,10] a current-driven type instability, but with oscillatory relative motion of ions and electrons. It has been shown (see for example Ref. 10) that a sinusoidal potential disturbance $\varphi_i(\mathbf{k},\omega)$ in the ion oscillatory (noninertial) frame of reference appears in the oscillating frame of reference associated with the electron component as a set of beats

$$\varphi_e(\mathbf{k},\omega)=\sum_{p=-\infty}^{\infty}J_p(a_{ie})e^{-ip\delta_{ie}}\varphi_i(\mathbf{k}-p\mathbf{k}_0,\omega-p\omega_0),$$

created by the sinusoidal plasma disturbance and harmonics of the pump wave. Inverse Landau damping of the electrons on these beats results in the excitation of kinetic parametric instabilities, including the ion cyclotron instability. In the case when the frequency ω_0 of the pump wave decreases to zero, the kinetic parametric instability smoothly transforms[9,10] into the kinetic ion cyclotron instability with transverse current.[12]

The second term, which contains the summation over indices m, n, s, arises from the dependence of the potential harmonics $\varphi_i(\mathbf{k}+m\mathbf{k}_{\|0},\omega+m\omega_0)$ on the potential $\varphi_i(\mathbf{k},\omega)$ through the relation described by Eq. (4). This interaction is responsible for the resonant decay of the pump wave into two waves with wave numbers and frequencies $\mathbf{k}$, $\omega(\mathbf{k})$ and $\mathbf{k}_1=\mathbf{k}+m\mathbf{k}_{0\|}$, $\omega_1=\omega(\mathbf{k}+m\mathbf{k}_{0\|})$ for which $\varepsilon[\mathbf{k},\omega(\mathbf{k})]=0$ and $\varepsilon[\mathbf{k}_1,\omega(\mathbf{k}_1)]=0$[13] or nonresonant quasimode decay, for which $\varepsilon[\mathbf{k},\omega(\mathbf{k})]=0$, but $\varepsilon[\mathbf{k}_1,\omega(\mathbf{k}_1)]\neq 0$. In the case of resonant decay, the potential $\varphi_i(\mathbf{k}+m\mathbf{k}_{\|0},\omega+m\omega_0)$ is also an eigenmode. For nonresonant decay, the potential $\varphi_i(\mathbf{k}+m\mathbf{k}_{\|0},\omega+m\omega_0)$ is defined as the quasimode.[7,8] Note that because of the Bessel function product $J_n(a_{ie})J_{n+m}(a_{ie})J_s(a_{ie})J_{s-m}(a_{ie})$, the growth rate of the quasimode decay instability for $a_{ie}\sim 1$ is less than the growth rate for the kinetic parametric instability. Keeping the largest terms, those with $J_0(a_{ie})$ and $J_1(a_{ie})$ Bessel functions, Eq. (6) becomes

$$\Big(1+\delta\varepsilon_i(\mathbf{k},\omega)+J_0^2(a_{ie})\,\delta\varepsilon_e(\mathbf{k},\omega)+J_1^2(a_{ie})$$
$$\times[\delta\varepsilon_e(\mathbf{k}-\mathbf{k}_{0\|},\omega-\omega_0)+\delta\varepsilon_e(\mathbf{k}+\mathbf{k}_{0\|},\omega+\omega_0)]$$
$$-J_0^2(a_{ie})J_1^2(a_{ie})\Bigg\{\frac{[\delta\varepsilon_e(\mathbf{k},\omega)-\delta\varepsilon_e(\mathbf{k}+\mathbf{k}_{0\|},\omega+\omega_0)]}{\varepsilon(\mathbf{k}+\mathbf{k}_{0\|},\omega+\omega_0)}$$
$$+\frac{[\delta\varepsilon_e(\mathbf{k},\omega)-\delta\varepsilon_e(\mathbf{k}-\mathbf{k}_{0\|},\omega-\omega_0)]}{\varepsilon(\mathbf{k}-\mathbf{k}_{0\|},\omega-\omega_0)}\Bigg\}\Big)\varphi_i(\mathbf{k},\omega)=0. \tag{7}$$

Equation (7) can be further simplified when only small displacements of ions relative to electrons or long wavelength modes, are considered, i.e., $a_{ie}\sim k\xi_{ie}\ll 1$.[7,8] In this asymptotic case, the growth rates of the kinetic parametric instability and the quasimode decay instability are proportional to a_{ie}^2 and are of the same order with respect to a_{ie}^2 [note, that for the analysis of the quasimode decay instabilities considered in Refs. 7 and 8, Eq. (7) has to be transformed by the appropriate changes of wave numbers and frequencies into the equation for the potential $\varphi_i(\mathbf{k}-\mathbf{k}_{\|0},\omega-\omega_0)$].

Since the primary objective of this paper is to estimate the effect of the ion cyclotron kinetic parametric instability[9,10] on the formation of the population of the suprathermal ions observed in the edge of the Alcator C-Mod tokamak, the remainder of the analysis in this work will focus on the first term in Eq. (6).

III. ION CYCLOTRON KINETIC PARAMETRIC TURBULENCE

Here we consider the linear and nonlinear evolution of the ion cyclotron kinetic parametric instability for plasma parameters consistent with cold edge plasmas in tokamak. The condition $|\delta\varepsilon_e|\ll|\delta\varepsilon_i|$ is automatically satisfied in plasmas where the temperature of cold ions is much less than the electron temperature. $T_e\gg T_i$ is observed in the cold edge plasma of the Alcator C-Mod tokamak.[2] The edge ions have a "two-temperature" structure with a noticeably higher effective temperature of the tail due to a population of suprathermal ions. Because of the low density and high temperature of the suprathermal ions, their affect on the dielectric permittivity $\varepsilon(\mathbf{k},\omega)$ is negligible and is omitted here. We assume that the rf pump electric field is such that the velocity of the oscillation of ions relative to the electrons in the wave field is less than the cold ion thermal velocity v_{Ti}. Keeping only zeroth and first order terms in the quantity $|\delta\varepsilon_e/\delta\varepsilon_i|<1$ in Eq. (5), we obtain

$$\varepsilon(\mathbf{k},\omega)\varphi_\alpha(\mathbf{k},\omega)=0. \tag{8}$$

The solution of the zeroth order equation

$$\{1+\delta\varepsilon_i[\mathbf{k},\omega(\mathbf{k})]\}\varphi_i(\mathbf{k},\omega)$$
$$=\left\{1+\frac{1}{k^2\lambda_{Di}^2}\left[1-\sum_{n=-\infty}^{\infty}\frac{\omega}{\omega-n\omega_{\mathrm{ci}}}\cdot I_n(k_\perp^2\rho_\alpha^2)e^{-k_\perp^2\rho_{L\alpha}^2}\right]\right\}$$
$$\times\varphi_i(\mathbf{k},\omega)=0,$$

has the form $\varphi_i(\mathbf{k},\omega)=\varphi_i(\mathbf{k})\delta[\omega-\omega(\mathbf{k})]$, where $\omega(\mathbf{k})$ is the frequency of ion cyclotron waves as determined from the solution of the equation $1+\delta\varepsilon_i[\mathbf{k},\omega(\mathbf{k})]=0$ and is equal to $\omega(\mathbf{k})=n\omega_{\mathrm{ci}}+\delta\omega(\mathbf{k})\approx n\omega_{\mathrm{ci}}[1+I_n(k_\perp^2\rho_i^2)e^{-k_\perp^2\rho_i^2}(1+k^2\lambda_{Di}^2]^{-1})$. In obtaining Eq. (8), we have also neglected the cyclotron damping of ion cyclotron waves on ions (i.e., $|\omega(\mathbf{k})-n\omega_{\mathrm{ci}}|\gg k_\| v_{\mathrm{Ti}}$).

From the first order terms in Eq. (5), we obtain the growth rate of the ion cyclotron kinetic parametric instability

$$\gamma(\mathbf{k})=-\left[\frac{\partial\,\mathrm{Re}\,\varepsilon}{\partial\omega(\mathbf{k})}\right]^{-1}\mathrm{Im}\sum_{\nu=-\infty}^{\infty}J_\nu^2(a_{ie})\,\delta\varepsilon_e$$
$$\times(\mathbf{k}-\nu\mathbf{k}_{\|0},\omega-\nu\omega_0)$$
$$\approx n\omega_{\mathrm{ci}}\left(\frac{\pi}{2}\right)^{1/2}\frac{T_i}{T_e}\frac{I_n(k_\perp^2\rho_i^2)e^{-k_\perp^2\rho_i^2}}{(1+k^2\lambda_{Di}^2)^2}$$
$$\times\sum_{\nu=-\infty}^{\infty}J_\nu^2(a_{ie})\frac{[\nu\omega_0-\omega(\mathbf{k})]}{|k_\|-\nu k_{0\|}|v_{\mathrm{Te}}}\cdot$$
$$\times\exp\left\{-\frac{[\omega(\mathbf{k})-\nu\omega_0]^2}{2(k_\|-\nu k_{0\|})^2v_{\mathrm{Te}}^2}\right\}. \tag{9}$$

Equation (9) is a generalization of the growth rate of the kinetic parametric instability to include a finite wavelength of the pump wave ($\mathbf{k}_{\|0}\neq 0$). Previous calculations (see Refs. 9 and 10), were restricted to the case of a dipole pump wave, $\mathbf{k}_{\|0}=0$. The kinetic parametric instability is driven by inverse Landau damping of electrons moving along the magnetic field in resonance with the beat wave formed by the unstable ion cyclotron wave $[\omega(\mathbf{k})]$ with harmonics $(\nu\omega_0)$ of the rf pump wave when $\nu\omega_0>\omega(\mathbf{k})$. For the kinetic parametric instability, all the $m\neq 0$, n, s terms in the sums in Eq. (6) are of the order of, or less than, $(\delta\varepsilon_e/\delta\varepsilon_i)^2\sim[\gamma/\omega(k)]^2\ll 1$ and may be neglected in the calculations.

For Alcator C-Mod tokamak plasma parameters ($n\sim 10^{13}$ cm^{-3}, ion temperature $T_i\geq 3$ eV, electron temperature $T_e=20$ eV, rf electric field $E_0\approx 1$ kV/cm, and confining magnetic field $B_0=4\times 10^4$ G), the maximum growth rate according to Eq. (9) in the radial region where $T_i\leq 10$ eV ($v_{\mathrm{Ti}}\leq 3\times 10^6$ cm/s, $u\approx cE_0/B_0\approx 2.5\times 10^6$ cm/s) occurs for $k_\perp\approx 230$ cm^{-1}. For this perpendicular wave number, $k_\perp\rho_i\approx 1.8$, $k\lambda_{Di}\approx 0.18$, and $k_\perp\xi\sim a_{ie}\approx 1.6$. In other words, the instability growth rate is largest for oscillations with wavelengths comparable to the displacement of ions relative to electrons in the rf pumping field.[9,10] Because the growth rate of the kinetic parametric instability is largest for $a_{ie}\sim 1$, the approximation $a_{ie}\ll 1$ used in Refs. 7 and 8 potentially ignores an entire class of strongly growing instabilities in Alcator C-Mod.

Numerical analysis of Eq. (9) indicates that, for pumping frequencies such that $\omega_0\geq\omega_{\mathrm{ci}}$, the dominant contribution to the growth rate comes from the term with $\nu=1$. The $\nu=0$, $\nu=-1$, and $|\nu|\geq 2$ terms have a negligible contribution and may be neglected. Note that even for $|\delta\varepsilon_i|\sim|\delta\varepsilon_e|\sim 1$, the $J_0^2(a_{ie})J_1^2(a_{ie})$ multiplier in Eq. (7) has a small numerical value, less than 0.05, and is ten times smaller then $J_1^2(a_{ie})$ for all values of a_{ie}. For large a_{ie}, $a_{ie}\sim 1.8-2.2$, the ratio of the quasimode decay multiplier to the kinetic parametric instability multiplier is even smaller. Therefore, all terms with $J_0^2(a_{ie})J_1^2(a_{ie})$ in Eq. (7) may be safely omitted in the analysis of the kinetic parametric instability. The difference in the Bessel function multipliers yields a growth rate for the kinetic parametric ion cyclotron instability, according to Eq. (9), that is ten times greater than the growth rate of the quasimode decay instabilities considered in Refs. 7 and 8.

To obtain the maximum instability growth rate, the condition for maximum inverse electron Landau damping of the beat wave must also be satisfied, i.e., $\nu\omega_0 - n\omega_{ci} \approx |k_\parallel - \nu k_{0\parallel}| v_{Te}$. Therefore, the maximum growth rate given by Eq. (9) may be estimated by

$$\gamma_{max} \sim \omega_{ci} \frac{u}{v_{Ti}} \frac{T_i}{T_e} J_1^2(a_{ie}) < \delta\omega(k) \sim \omega_{ci} \frac{u}{v_{Ti}}, \tag{10}$$

where, because $a_{ie} \sim 1$, the approximation $k\rho_i \sim v_{Ti}/u$ has been used. It is interesting to note that this kinetic instability exists even for $k_\parallel = 0$ if the condition $|\nu\omega_0 - \omega(\mathbf{k})| \approx |\nu v_{Te} k_{0\parallel}|$ is fulfilled for some integer ν.

Because the growth rate of kinetic parametric instability is less than ion cyclotron wave frequency shift, i.e., $|\gamma| \ll |\omega(\mathbf{k}) - n\omega_{ci}|$, and the wave number spectrum $\Delta k_\perp$ of unstable waves is wide, i.e., $\Delta k_\perp \sim k_\perp$, we can use the theory of weak kinetic parametric turbulence[10] in the analysis of the nonlinear stage and saturation of instability. The basic equation of that theory, which governs the weak nonlinear evolution and saturation of kinetic parametric ion cyclotron instabilities in the random phase approximation, is provided in the Appendix as Eq. (A1). Equation (A1) describes the temporal evolution of the spectral intensity of the unstable ion cyclotron waves $I(\mathbf{k},t)$. Essentially, Eq. (A1) accounts for the finite displacement of electrons with respect to ions in the pumping field $\mathbf{E}_0(\mathbf{r},t) = \mathbf{E}_0 \cos(\omega_0 t - \mathbf{k}_{0\parallel}\mathbf{r})$. Analysis of matrix elements v_α and w_α in Eq. (A1) demonstrates that the principle nonlinear process, which dominates the nonlinear evolution of the kinetic parametric instability in the short wave length limit, $k_\perp \rho_i \gg 1$, is the induced scattering of ion cyclotron waves on bare ions. In the short wave length limit, the matrix element that defines this process is[12]

$$\begin{aligned} w_i[\mathbf{k},\omega(\mathbf{k})|\mathbf{k}_1,\omega(\mathbf{k}_1)|\mathbf{k},\omega(\mathbf{k})] &\approx -\sqrt{\frac{2}{\pi}} \frac{1}{k^2\lambda_{Di}^2} \frac{e^2}{T_i^2} k_\perp \rho_i k_{i\perp} \rho_i \\ &\times [\sin^2(\theta - \theta_1) \ln k_\perp \rho_i + O(1)] \frac{\omega_{ci}^2}{\delta\omega(\mathbf{k})} \\ &\times \left[\frac{n\omega_{ci}}{\delta\omega(\mathbf{k})} - \frac{n_1\omega_{ci}}{\delta\omega_1(\mathbf{k}_1)} \right] \delta[\delta\omega(\mathbf{k}) - \delta\omega_1(\mathbf{k}_1)]. \end{aligned} \tag{11}$$

From the balance equation $\gamma(\mathbf{k}) + \Gamma(\mathbf{k}) = 0$, where $\Gamma(\mathbf{k})$ is determined by Eq. (A2), we find that the linear growth rate equals the nonlinear growth rate at a turbulence level equal to

$$\frac{W}{n_i T_i} \sim \frac{T_i}{T_e} \left(\frac{u}{v_{Ti}} \right)^4, \tag{12}$$

where $W = \int d\mathbf{k} W(\mathbf{k}) = (1/4\pi) \int d\mathbf{k} I(\mathbf{k}) k^2 \omega(\mathbf{k}) \partial\varepsilon_i / \partial\omega(\mathbf{k})$ is the energy density of the ion cyclotron turbulence.

For the turbulence level given by Eq. (12), the matrix element given by Eq. (11) predicts that all ion cyclotron modes at frequencies $\omega \approx n\omega_{ci}$ with $n \geqslant 2$ are suppressed. Only the fundamental ion cyclotron mode with ion cyclotron frequency $\omega \approx \omega_{ci}$ survives. The saturation of this unstable ion cyclotron mode results from the nonlinear broadening of the ion cyclotron resonance governed by the energy density level of ion cyclotron turbulence; given by the estimate[10,14]

$$\frac{W}{n_i T_i} \sim \left(\frac{u}{v_{Ti}} \right)^4. \tag{13}$$

The estimated energy density level of ion cyclotron turbulence, Eq. (13), decreases rapidly for $u \ll v_{Ti}$. Thus, this turbulence may be important in the plasma edge where the conditions of cold ions and strong pumping fields are both satisfied near the antenna. For comparable values of u and v_{Ti}, i.e., when the wavelength of the most unstable ion cyclotron waves is comparable with ion Larmor radius, $k_\perp \rho_i \sim 1$, the energy density of the ion cyclotron turbulence in the saturated state may be quite large and the ion cyclotron turbulence may be the largest component of the energy stored in the cold edge plasma. From the perspective of maximizing the energy density of the turbulence in the saturated state, the condition $u \sim v_{Ti}$ may be considered as a threshold condition at which the ion cyclotron kinetic parametric turbulence and any effects on the plasma are observable. However, for $k_\perp \rho_i \sim 1$, the induced scattering of ion cyclotron waves by electrons and by the polarization clouds of virtual waves, as well as the decay of the ion cyclotron waves, are of the same order as the induced scattering of ion cyclotron waves on bare ions. Therefore, all possible nonlinear saturation processes must be considered through numerical solutions of Eqs. (A1) and (A2). It is worth noting that for the case of $T_i \sim T_e$, as seen in the edge plasmas of the DIII-D tokamak, the condition $|\gamma| \ll |\omega(\mathbf{k}) - n\omega_{ci}|$ for validity of the weak turbulence theory of ion cyclotron turbulence is only satisfied because of the smallness of multiplier $J_1^2(a_{ie})$; maximum value is 0.34 at $a_{ie} = 1.8$. However even for the $T_i \sim T_e$, the saturated state arises from the nonlinear broadening of the ion cyclotron resonance and the level of ion cyclotron turbulence can still be estimated by Eq. (13).

Development of ion cyclotron kinetic parametric turbulence leads to the turbulent heating of cold edge ions and electrons. To estimate the heating rate of cold, nonresonant, edge ions we begin with the quasilinear equation in which the random walk of ions in the fields of the ion cyclotron turbulence is included

$$\begin{aligned} \frac{\partial F_{oi}}{\partial t} &\approx \frac{e_i^2}{m_i^2} \frac{n\omega_{ci}}{v_{i\perp}} \frac{\partial}{\partial v_{i\perp}} \\ &\times \int d\mathbf{k} I(\mathbf{k}) J_n^2\left(\frac{k_\perp v_{\perp i}}{\omega_{ci}} \right) \frac{\nu_i}{\delta\omega^2 + \nu_i^2} \frac{1}{v_{i\perp}} \frac{\partial F_{oi}}{\partial v_{i\perp}}, \end{aligned} \tag{14}$$

where the resonance broadening term ν_i was determined in Ref. 15. Multiplying Eq. (14) by $m_i v_{i\perp}^2/2$ and integrating over velocities $\mathbf{v}_i$ we obtain the expressions[10]

$$n_i \frac{\partial T_{i\perp}}{\partial t} \sim n_e \frac{\partial T_e}{\partial t} \sim \nu_i W \sim \gamma W \sim \omega_{\rm ci} \frac{T_i}{T_e} \left(\frac{u}{v_{\rm Ti}} \right)^5 n_i T_i . \quad (15)$$

From the terms in Eq. (15), it is instructive to calculate the effective collision frequency for absorption of the pump wave due to the development of ion cyclotron turbulence and turbulent heating of ions and electrons. From the balance equation

$$\nu_{\rm eff}\, n_i \frac{m_i u^2}{4} \sim n_i \frac{\partial T_{i\perp}}{\partial t},$$

where u is the amplitude of the ion quiver velocity in the pumping wave, we obtain

$$\nu_{\rm eff} \sim \omega_{\rm ci} \frac{T_i}{T_e} \left(\frac{u}{v_{\rm Ti}} \right)^3 . \quad (16)$$

For $u \sim v_{\rm Ti}$, the absorption of pump wave energy and turbulent ion heating occur in times comparable to the ion cyclotron period. Therefore, stochastic motion of ions in the fields of the ion cyclotron parametric turbulence is the primary mechanism for heating of cold edge ions. The parametrically unstable ion cyclotron waves may propagate with group velocity of the order of or less than the ion thermal velocity and can deliver the energy of the ion cyclotron turbulence into the inner plasma regions. Because the energy density of the parametric turbulence is not more than $nT_{i(\rm cold)}$, we estimate that the deposition of such a modest amount of energy into the inner plasma, where the temperatures are thousands of times larger, is a negligible effect.

IV. EDGE HEATING OF SUPRATHERMAL IONS IN THE ALCATOR C-MOD TOKAMAK BY ION CYCLOTRON KINETIC PARAMETRIC TURBULENCE

In the Alcator C-Mod tokamak, production of fast deuterium ions with energies more than 5 keV was observed in the scrape off layer with a neutral particle analyzer during hydrogen minority heating for rf power levels above approximately 500 kW.[2] The experimental observations indicated that the edge ion heating occurs for rf fields near the antenna of roughly $E_0 \approx 1$ kV/cm in the poloidal direction. In such a rf electric field and a confining magnetic field of $B_0 = 4\times 10^4$ G, the amplitude of the ion quiver velocity is $u \approx cE_0/B_0 = 2.5\times 10^6$ cm/s. In the scrape-off layer region, the ion temperature is estimated to be $T_i \geqslant 3$ eV and increases to $T_i = 1-4$ keV in the plasma center. For deuterium ions of temperature $T_i = 3$ eV, the thermal velocity v_{TD} is equal to $v_{TD} = 1.4\times 10^6$ cm/s$>u$ and the condition $u \approx v_{\rm Ti}$ is satisfied for deuterium ions with temperature $T_i \sim 10$ eV, i.e., deuterium ions closer to the center of the plasma. For plasma parameters, for which $v_{TD} \approx u$ the energy density level of ion cyclotron kinetic parametric turbulence predicted by Eq. (13) is high, $W \sim n_i T_i$. In Ref. 2, the 500 kW power threshold was assumed to correspond to the electric field threshold required for parametric decay into ion Bernstein wave and an ion cyclotron quasimode. Equation (10) predicts that the parametric kinetic instability may be excited in much weaker pump electric fields. However, the resulting modest level of ion cyclotron turbulence at threshold may not be detectable. Moreover, for the experimental conditions in the edge of the Alcator C-Mod tokamak,[2] the most unstable ion cyclotron waves are predicted to have wavelengths on the order of $\rho_i \sim 10^{-2}$ cm. The probes currently in use in Alcator C-Mod cannot detect electrostatic waves with such short wavelength.

A second threshold discussed in Ref. 2 concerns the value of the longitudinal component of unstable ion cyclotron waves required for the parametric decay into an ion Bernstein wave and ion cyclotron quasimode, $k_\parallel = 0.22$ cm$^{-1} \sim k_{0\parallel}$. For $k_\parallel = 0.22$ cm^{-1}, the parallel wavelength of the unstable waves is so long that the toroidal field varies significantly over one wavelength—thereby violating the uniform plasma assumption in the theory. For the parametric kinetic instability considered here, the constraints on the value of $k_\parallel$ are less restrictive. For $\omega_{cD} = 2\times 10^8$ s^{-1}, $\omega_0 = 2.7\omega_{cD} = 5.4\times 10^8$ s^{-1}, and $T_e = 20$ eV,[2] $k_\parallel$ for the most unstable ion cyclotron waves is $k_\parallel \approx 1.7\omega_{cD}/v_{Te} = 1.3$ cm$^{-1} \gg k_{0\parallel}$ and the value of the longitudinal component of the pump wave field, $k_{\parallel 0}$, for Alcator C-Mod experimental conditions does not appear in the theory.

The ion distribution function in edge of these tokamak plasmas may be characterized as a superposition of two groups of ions: a group of cold ions that determine the dispersion relation of ion cyclotron waves, and smaller ''passive'' group of ions which because of low density and high temperature do not participate in the generation of the ion cyclotron turbulence, but may experience the heating by the ion cyclotron turbulence. The appearance of the suprathermal ions does not result in the heating of the bulk cold ions or the reduction of the turbulence level (13) because the ion–ion collision frequency is very small and high-energy ions leave the plasma without collisional dissipation of their energy.

As noted earlier, for Alcator C-Mod tokamak experimental conditions, ion cyclotron waves generated by kinetic parametric ion cyclotron turbulence have short wavelengths along the magnetic field, $k_\parallel \sim 1-2$ cm^{-1}, and have wavelengths much smaller then the wavelength of the electromagnetic pump wave. Therefore, it is conceivable that a strong interaction of the ion cyclotron turbulence with hot ions at the conditions of the ion cyclotron resonance might occur. The energy absorption rate by the hot (suprathermal) ions is proportional to the imaginary part of their partial dielectric permittivity, $\mathrm{Im}\,\varepsilon_{i(\rm hot)}$, and is a maximum for $(\omega - \omega_{\rm ci})/\sqrt{2} k_\parallel v_{\rm Ti(hot)} \sim 1$. For deuterium ions in Alcator C-Mod with energy 5 keV and $k_\perp \rho_{i(\rm cold)} \sim 1$, $k_\parallel \sim 1-2$ cm^{-1} and $(\omega - \omega_{\rm ci})/\sqrt{2} k_\parallel v_{\rm Ti(hot)} \sim 1-2$.[2] For hydrogen ions with the same energy and for the same wavelength, $(\omega - \omega_{\rm ci})/\sqrt{2} k_\parallel v_{\rm Ti(hot)} \sim 1.5-3$. It is important to note that in this work, we do not consider the problem of how the population of suprathermal ions, the ''tail,'' originally arises. We only consider whether it is possible to sustain the population of suprathermal ions by interactions with ion cyclotron parametric turbulence. To answer this question, the turbulent heating rate of hot ions in the plasma edge must be determined. The turbulent ion heating rate is obtained from the quasilinear equation for the distribution function of the population of hot ions in the cold edge

$$\frac{\partial F_{i(\mathrm{hot})}}{\partial t}=\frac{e^2}{m_i^2}\int d\mathbf{k}I(\mathbf{k})\left(\frac{\omega_{\mathrm{ci}}}{v_\perp}\frac{\partial}{\partial v_\perp}+k_\parallel\frac{\partial}{\partial v_\parallel}\right)\left[\frac{\omega_{\mathrm{ci}}}{v_\perp}\frac{\partial F_{i(\mathrm{hot})}}{\partial v_\perp}+k_\parallel\frac{\partial F_{i(\mathrm{hot})}}{\partial v_\parallel}\right]\times J_1^2\left(\frac{k_\perp v_\perp}{\omega_{\mathrm{ci}}}\right)\delta(\omega-\omega_{\mathrm{ci}}-k_\parallel v_\parallel). \tag{17}$$

Multiplying Eq. (17) by $m_i v_i^2/2$ and integrating over velocities $\mathbf{v}_i$, we obtain the following estimate for an upper bound on the suprathermal ion heating rate at ion cyclotron resonance

$$\frac{\partial T_{i(\mathrm{hot})}}{\partial t}=-\int d\mathbf{k}W(\mathbf{k})\frac{\mathrm{Im}\,\varepsilon_{i(\mathrm{hot})}}{\dfrac{\partial\,\mathrm{Re}\,\varepsilon_{i(\mathrm{cold})}}{\partial\omega(k)}}\leqslant\omega_{\mathrm{ci}}\left[\frac{T_{i(\mathrm{cold})}}{T_{i(\mathrm{hot})}}\right]^{5/2}T_{i(\mathrm{hot})}, \tag{18}$$

where the level of turbulence is determined by Eq. (13). The heating rate predicted by Eq. (18) is quite small, even for parametric ion cyclotron turbulence with the largest growth rate and resulting largest level of turbulence, i.e., $k_\perp\rho_{i(\mathrm{cold})}\sim 1$. Therefore, ion cyclotron parametric turbulence cannot be responsible for the generation of suprathermal ions nor can such turbulence increase the energy of the suprathermal ions in any significant way.

V. CONCLUSIONS

Parametrically excited waves have been observed in several tokamak experiments during ion cyclotron resonance heating.[1–6] Typical experimental results indicate that the decay waves have modest amplitudes and therefore these waves do not deposit significant power into the edge plasma.[16] The correlation between the appearance of a population of suprathermal ions in the edge plasma and the appearance of parametrically excited waves simultaneously in the region near the ICRF antenna plasma has led a number of researchers to suggest that parametric turbulence is responsible for the generation of the suprathermal (hot) ions.[1–4] The principle aim of this work was to determine the heating rate of suprathermal ions due to their interaction with ion cyclotron parametric turbulence in regions near the ICRF antenna. We examined the linear and nonlinear stages of the ion cyclotron kinetic parametric instability, which has a growth rate significantly larger than the growth rate of the quasimode decay instabilities considered in Refs. 7 and 8. We found that interaction of cold edge ions with parametric ion cyclotron turbulence leads to significant stochastic heating of the cold ions due to their random walks in the fields of the ion cyclotron turbulence according to Eq. (15). At the same time, we found that even at the upper bound for the ion heating rate, Eq. (18), the predicted ion heating rate is too small to explain the observed rapid energy gain of suprathermal ions in the edge of tokamak plasmas. Therefore, it is unlikely the parametric ion cyclotron instabilities described by the linear theory of Refs. 7–9, are responsible for the formation of a population of suprathermal ions in these experiments.

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APPENDIX: WEAK PARAMETRIC TURBULENCE OF A PLASMA IN A STRONG PUMP WAVE

The nonlinear evolution and saturation of the kinetic parametric ion cyclotron instability is considered in the random phase approximation on the basis of the equation for the spectral intensity of the unstable ion cyclotron waves $I(\mathbf{k},t)$ as determined by

$$\langle\varphi_i(\mathbf{k})\varphi_i(\mathbf{k}')\rangle=I(\mathbf{k})\cdot\delta(\mathbf{k}+\mathbf{k}')\,\delta(\omega+\omega')\,\delta[\omega-\omega(\mathbf{k})]. \tag{A1}$$

That equation, in which all weak nonlinear processes are included is given by[10]

$$\begin{aligned}\frac{1}{2}\frac{\partial I(\mathbf{k},t)}{\partial t}=&\gamma(\mathbf{k})I(\mathbf{k},t)+\Gamma(\mathbf{k},t)I(\mathbf{k},t)-\pi\sum_{\alpha,\beta=i,e}\sum{}'\ \mathrm{Re}\int d\mathbf{k}_1 v_\alpha[\mathbf{K},\Omega(\mathbf{K})|\mathbf{K}_1,\Omega_1(\mathbf{K}_1)]\\&\times v_\beta[\mathbf{K}_2',\Omega_2'(\mathbf{K}_2')|\mathbf{K}',\Omega'(\mathbf{K}')]A^{(\alpha)}_{\nu,p,q}A^{(\alpha)*}_{\nu_1,p_1,q_1}\left\{\frac{\partial\,\mathrm{Re}\,\varepsilon_i[\mathbf{k},\omega(\mathbf{k})]}{\partial\omega(\mathbf{k})}\frac{\partial\,\mathrm{Re}\,\varepsilon_i[\mathbf{k}_2,\omega_2(\mathbf{k}_2)]}{\partial\omega_2(\mathbf{k}_2)}\right\}^{-1}\\&\times I(\mathbf{k})\cdot I(\mathbf{k}_1)\,\delta[\Omega(\mathbf{K})-\Omega_1(\mathbf{K}_1)-\Omega_2(\mathbf{K}_2)]+\frac{\pi}{2}\sum_{\alpha,\beta=i,e}\sum{}'\ \mathrm{Re}\int d\mathbf{k}_1 v_\alpha[\mathbf{K},\Omega(\mathbf{K})|\mathbf{K}_1,\Omega_1(\mathbf{K}_1)]\\&\times A^{(\alpha)}_{\nu,p,q}A^{(\alpha)*}_{\nu_1,p_1,q_1}\cdot v^*_\beta[\mathbf{K}',\Omega'(\mathbf{K}')|\mathbf{K}_1',\Omega_1'(\mathbf{K}_1')]\left\{\frac{\partial\,\mathrm{Re}\,\varepsilon_i[\mathbf{k},\omega(\mathbf{k})]}{\partial\omega(\mathbf{k})}\right\}^{-2}I(\mathbf{k}_1)I(\mathbf{k}_2)\\&\times\delta[\Omega(\mathbf{K})-\Omega_1(\mathbf{K}_1)-\Omega_2(\mathbf{K}_2)].\end{aligned} \tag{A2}$$

The symbol Σ' means the summation over v,p,q,v_1,p_1,q_1 from $-\infty$ to $+\infty$ under the condition $v-p-q=v_1-p_1-q_1$. In Eq. (A2) $\gamma(\mathbf{k})$ is linear growth rate determined by Eq. (9), $\Gamma(\mathbf{k})$ is nonlinear growth rate (damping), which is determined by the equation

$$\begin{aligned}\Gamma(\mathbf{k})=&\sum_{\alpha=i,e}\sum{}''\left[\frac{\partial\,\mathrm{Re}\,\varepsilon_i}{\partial\omega(\mathbf{k})}\right]^{-1}\int d\mathbf{k}_1 B^{(\alpha)}_{\nu,p,q,r} I(\mathbf{k}_1)\, w_\alpha[\mathbf{K},\Omega(\mathbf{K})|\mathbf{K}_1,\Omega_1(\mathbf{K}_1)|\mathbf{K}+(\nu-q)\mathbf{k}_{0\parallel},\Omega(\mathbf{K})++(\nu-q)\omega_0]\\
&+\sum_{\alpha,\beta=i,e}\sum{}'\ \mathrm{Im}\int d\mathbf{k}_1 A^{(\alpha)}_{\nu,p,q}A^{(\alpha)*}_{\nu_1,p_1,q_1}\,\upsilon_\alpha[\mathbf{K},\Omega(\mathbf{K})|\mathbf{K}_1,\Omega_1(\mathbf{K}_1)]\left\{\frac{\partial\,\mathrm{Re}\,\varepsilon_i[\mathbf{k},\omega(\mathbf{k})]}{\partial\omega(\mathbf{k})}\right\}^{-1}\cdot\upsilon_\beta[\mathbf{K}'_2,\Omega'(\mathbf{K}')\\
&-\Omega'_1(\mathbf{K}'_1)|\mathbf{K}',\Omega'(\mathbf{K}')]I(\mathbf{k}_1)\Bigg\{1+\delta\varepsilon_\alpha[\mathbf{K}-\mathbf{K}_1+q\mathbf{k}_{0\parallel},\Omega(\mathbf{K})-\Omega_1(\mathbf{K}_1)+q\omega_0]\\
&+\sum_{m=-\infty}^{\infty}J_m^2(a_{\alpha\beta})\,\varepsilon_\beta[\mathbf{K}_2+(q-m)\mathbf{k}_{0\parallel},\Omega(\mathbf{K})-\Omega_1(\mathbf{K}_1)+(q-m)\omega_0]\Bigg\}^{-1}.\end{aligned}\tag{A3}$$

In Eq. (A3), the symbol Σ'' means a summation over ν,p,q,r from $-\infty$ to $+\infty$ under the condition $\nu+r=p+q$. In Eqs. (A2) and (A3), we employ the following notation:

$$\begin{aligned}&\upsilon_\alpha[\mathbf{K},\Omega(\mathbf{K})|\mathbf{K}_1,\Omega_1(\mathbf{K}_1)]=V_\alpha[\mathbf{K},\Omega(\mathbf{K})|\mathbf{K}_1,\Omega_1(\mathbf{K}_1)]+V_\alpha[\mathbf{K},\Omega(\mathbf{K})|\mathbf{K}-\mathbf{K}_1,\Omega(\mathbf{K})-\Omega_1(\mathbf{K}_1)],\\
&w_\alpha[\mathbf{K},\Omega(\mathbf{K})|\mathbf{K}_1,\Omega_1(\mathbf{K}_1)|\mathbf{K}+(\nu-q)\mathbf{k}_{0\parallel},\Omega(\mathbf{K})+(\nu-q)\omega_0]\\
&\quad=W_\alpha[\mathbf{K},\Omega(\mathbf{K})|\mathbf{K}_1,\Omega_1(\mathbf{K}_1)|\mathbf{K}+(\nu-q)\mathbf{k}_{0\parallel},\Omega(\mathbf{K})+(\nu-q)\omega_0]\\
&\qquad+W_\alpha[\mathbf{K},\Omega(\mathbf{K})|\mathbf{K}_1,\Omega_1(\mathbf{K}_1)|-\mathbf{K}_1-(\nu-q)\mathbf{k}_{0\parallel},\Omega_1(-\mathbf{K}_1)-(\nu-q)\omega_0],\\
&\mathbf{K}=\mathbf{k}-\nu\mathbf{k}_{0\parallel},\quad \mathbf{K}_1=\mathbf{k}_1-p\mathbf{k}_{0\parallel},\quad \mathbf{K}_2=\mathbf{k}_2-q\mathbf{k}_{0\parallel},\quad \mathbf{K}'=\mathbf{k}-\nu_1\mathbf{k}_{0\parallel},\quad \mathbf{K}'_1=\mathbf{k}'_1-p_1\mathbf{k}_{0\parallel},\\
&\mathbf{K}_2=\mathbf{k}_2-q_1\mathbf{k}_{0\parallel},\quad A^{(i)}_{\nu,p,q}=\delta_{\nu0}\delta_{p0}\delta_{q0},\quad A^{(e)}_{\nu,p,q}=J_\nu(a_{ie})J_p(a_{1ie})J_q(a_{2ie})e^{ip\delta_1+iq\delta_2-i\nu\delta},\\
&B^{(i)}_{\nu,p,q,r}=\delta_{\nu0}\delta_{p0}\delta_{q0}\delta_{r0},\quad B^{(e)}_{\nu,p,q,r}=J_\nu(a_{ie})J_p(a_{1ie})J_q(a_{ie})J_r(a_{1ie})e^{-i\nu\delta+ip\delta_1+iq\delta-ir\delta_1},\end{aligned}$$

where $\delta_{\nu0}$ and others are the Kronecker delta. Equation (A2) differs from the equation for spectral intensity $I(\mathbf{k},t)$ in usual theory of weak turbulence (see, for example, Ref. 12) by the factors $A^{(e)}$ and $B^{(e)}$ that account for the finite displacement of electrons with respect to ions in the pumping field and the presence of shifts in the frequencies ω, ω_1, and ω_2 from harmonics of the pumping wave frequency. In analogy with the ordinary kinetic equation for waves, it may be concluded that third and forth terms on the right-hand side of Eq. (A2) describe decay processes involving of waves (or beat waves) $[\mathbf{K},\Omega(\mathbf{K})]$, $[\mathbf{K}_1,\Omega_1(\mathbf{K}_1)]$, and $[\mathbf{K}_2,\Omega_2(\mathbf{K}_2)]$. The nonlinear growth rate $\Gamma(\mathbf{k})$ describes the processes of induced scattering of waves, or their beat waves, with harmonics of the pump wave by bare particles [the first term on the right-hand side of Eq. (A3)] and by the polarization clouds of virtual waves $[\mathbf{K}_2+q\mathbf{k}_{0\parallel},\Omega(\mathbf{K})-\Omega_1(\mathbf{K}_1)+q\omega_0]$ [the second term in Eq. (A3)]. The matrix elements υ_α and w_α $(\alpha=i,e)$ are given by expressions (see, for example, Ref. 12) previously obtained for magnetized plasmas without a pump wave field and now modified to include changes in the wave numbers and wave frequencies. The nonlinear terms in Eq. (A2) are of the same order as of the growth rate $\gamma(\mathbf{k})$. Therefore, omitting terms of order of $\gamma(\mathbf{k})[\gamma(\mathbf{k})/\omega(\mathbf{k})]$ [i.e., the terms of the order of $(\delta\varepsilon_e/\delta\varepsilon_i)^2\ll 1$], in the Eq. (7) is justified for the parametric kinetic instability. It is interesting to note that all the ion matrix elements enter into Eqs. (A2) and (A3) in the same form as in the case when the pump wave is absent. Note also that, as follows from the expressions required to write the Fourier transformed electrostatic potential evaluated in the laboratory $[\varphi(\mathbf{k},\omega)]$ in a coordinate system moving with the ions $[\varphi_i(\mathbf{k},\omega)]$:[10]

$$\varphi(\mathbf{k},\omega)=\sum_{m=-\infty}^{\infty}J_m(a_i)e^{im\delta_i}\varphi_i(\mathbf{k}+m\mathbf{k}_{0\parallel},\omega+m\omega_0),\tag{A4}$$

where[7–10]

$$a_\alpha=\frac{e_\alpha}{m_\alpha}\left[\left(\frac{\mathbf{E}_\perp\cdot\mathbf{k}_\perp}{\omega_0^2-\omega_{c\alpha}^2}+\frac{E_\parallel k_\parallel}{\omega_0^2}\right)^2+\frac{\omega_{c\alpha}^2|\mathbf{E}_\perp\times\mathbf{k}_\perp|^2}{(\omega_0^2-\omega_{c\alpha}^2)^2\omega_0^2}\right]^{1/2},$$

and $\mathbf{E}_\perp$ and $E_\parallel$ refer to the components of the pump wave perpendicular and parallel to the magnetic field, the spectral intensity $I(\mathbf{k},t)$ of oscillations given by potential φ_i is equal to the spectral intensity of the oscillations in the laboratory system of coordinates given by potential $\varphi(\mathbf{k},\omega)$.

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