WEST VIRGINIA UNIVERSITY

PLASMA PHYSICS GROUP

INTERNAL REPORT PL - 045

Mean Optical Depth and Optical Escape Factor for Helium Transitions in Helicon Plasmas

R.F. Boivin

November 2000
(Revised March 2001)

TABLE OF CONTENT

1.0 Introduction

Emission and absorption processes in plasmas are complex matters. More specifically, line intensities associated with transitions of plasma species (neutral or ion) are the result of spontaneous emission, stimulated emission and absorption. In order to use line intensity for any diagnostic, one must first examine if the plasma optically affects this specific transition. A line is optically thin if both stimulated emission and absorption are negligible compare to spontaneous emission. In this case, the line intensity $I(n)$ of that transition is given by:

$$I(n) = \quad (4p)^{-1} \text{ò} \; N_k \; A_k \; dV \qquad (1)$$

where N_k is the population of the excited population responsible for the transition, A_k is the spontaneous transition probability associated with that specific transition and dV is the unit plasma volume. The integration is carried out over the spectroscopy defined volume [1]. The plasma is optically thick (or opaque), if the absorption process is dominant and that only a small fraction of the emitted light escapes the plasma. The plasma is often optically thin for low-density plasmas ($\leq 10^{10}$ cm^{-3}) and becomes increasingly opaque as density increase. Generally, the plasma is opaque to resonance lines (transitions involving the ground state or metastables) but remains optically thin for transitions between excited levels even at moderately high density ($\geq 10^{14}$ cm^{-3}). The two important parameters to quantify the opacity of the plasma are the Optical Escape Factor (OEF) and the Mean Optical Depth (MOD). The OEF corresponds to the fraction of light that can escape the plasma; for OEF = 1 the plasma is optically thin. The MOD is the exponential absorption coefficient associated to a given transition. A MOD smaller or equal than .01 corresponds to optically thin plasmas.

2.0 Light absorption in plasma

The change of line intensity $I\ (n)$ related to absorption can be written as:

$$dI\ (n) = -\ I\ (n)\ c\ (n)\ r\ ds = -\ I\ (n)\ dt \qquad (2)$$

where $c\ (n)$ is the atomic absorption coefficient (units cm^2), r is the density (cm^{-3}), ds is the unit travel distance in the absorbing medium and $t(n)$ is the optical depth or the optical thickness of the medium (unitless).

$$t(n) = \text{ò}\ c\ (n)\ r\ ds \qquad (3)$$

Alternatively, $c\ (n)\ r\ ds$ can be written as $a\ (n)\ ds$, where $a\ (n)$ (cm^{-1}) is the absorption coefficient of the medium. Re-writing equation (2) in terms we have:

$$dI\ (n)\ /\ I\ (n) = -\ dt \qquad (4)$$

The solution of this differential equation is:

$$I\ (n) = I_o\ exp\ (-t\ (n)) = I_o\ exp\ (-a\ (n)\ x) \qquad (5)$$

Where I_o is the non-attenuated line intensity and x the absorbing medium length. Thus, equation (5) describe the intensity of a light beam of initial intensity I_o going through an absorbing medium.

3.0 Absorption and Emission in plasmas

Unfortunately, for a plasma which is both an emitting and absorbing medium the situation is more complex. Equation (2) must be re-written to account for that situation and we have [2]:

$$dI\,(n) = -\,I\,(n)\;c\,(n)\;r\;ds \;+\; J\,(n)\;r\;ds \qquad (6)$$

where $J(n)\;r\;ds$ is the source term of the new equation. Dividing each side of the equation $dt = c\,(n)\;r\;ds$ equation (6) becomes:

$$dI\,(n)\,/\,dt = -\,I\,(n) \;+\; J\,(n)\,/\,c\,(n) \qquad (7)$$

Equation (7) is the radiative transfer equation. The solution of this differential equation is given by:

$$\bar{I}(n) = \frac{J(n)}{c(n)}\left[1 - \exp\{-c(n)\,r\;\bar{D}\right] \qquad (8)$$

Where $\bar{I}(n)$ averaged intensity and $\bar{D}$ is a characteristic length of the plasma (depends on the geometry, ex: at the center of a sphere, $\bar{D}$ is the radius). Considering the spectrum region near the transition between level 1 (lower level; ground or excited) and level 2 (higher excited level) J (ν) can be written as:

$$J(n) = \frac{n_2\;f_2(n)\;hn}{r}\left(\frac{A_{21}}{4p} + \frac{I(n)B_{21}}{c}\right) \qquad (9)$$

where n_2 is the higher level population, $f_2(n)$ is the shape function of the emission line, A_{21} is the spontaneous emission probability and B_{21} is the transition probability of induced emission. Since the emission is not coherent, the stimulated emission term can be neglected and $J(n)$ becomes:

$$J(n) = \frac{n_2\;A_{21}\,f_2(n)\;hn}{4pr} \qquad (10)$$

Meanwhile, the atomic absorption coefficient $c\,(n)$ can be written as:

$$c(n) = \frac{n_1\;B_{12}\,f_1(n)\;hn}{r\;c} \qquad (11)$$

where B_{12} is the transition probability of absorption, $f_1(n)$ is the shape function governing the absorption and n_1 is the population of the lower level. We will now suppose that both emission and absorption profiles are dominated by Doppler broadening [3].

$$f(n) = f_1(n) = f_2(n) = \frac{1}{\sqrt{p}\;\Delta n_D}\exp\left[-\frac{(n - n_0)^2}{\Delta n_D^{\;2}}\right] \qquad (12)$$

where the Doppler frequency $Dn_D = (2kT/Mc^2)^{1/2}\;n_o = v_{th}\,n_o/c$, while n_o is the central frequency and v_{th} is the thermal velocity of the gas $(2kT/M)^{1/2}$. The average intensity can be written as:

$$\bar{I}(n) = \frac{c\ n_2\ A_{21}}{4p\ n_1\ B_{12}}\left[1 - \exp(-c(n)\,r\ \overline{D})\right] \tag{13}$$

Equation (13) describe the average intensity taking account both the emission and absorption processes in the plasma. Neglecting stimulated emission, the population of the excited level is given by:

$$\frac{dn_2}{dt} = -n_2 A_{21} + \frac{4p\ n_1 B_{12}}{c}\int \bar{I}(n)\,f(n)\,dn \qquad + \quad \textit{electron collisional and radiative transfer terms} \tag{14}$$

where the two first terms are related to the spontaneous emission and to the re-absorption, respectively. The other terms are associated to the electron impact excitation from ground state (only term for the Corona model) and secondary processes such electron impact from metastables, excitation transfer processes from neighboring levels etc… (Collisional Radiative model). Only the first two terms are involved in the emission and absorption of light emitted by the plasma. Re-writing equation (14) we have:

$$\frac{dn_2}{dt} = -n_2 A_{21}\left(1 - \frac{4p\ n_1\ B_{12}}{n_2 A_{21}\ c}\right)\int \bar{I}(n)\,f(n)\,dn \quad + \quad \textit{electron collisional and radiative transfer terms} \tag{15}$$

4.0 Optical Escape Factor and Mean Optical Depth

Introducing the Optical Escape Factor, equation (15) can be written as:

$$\frac{dn_2}{dt} = -\ n_2\ A_{21}\ \Lambda(t_o) \quad + \quad \textit{electron collisional and radiative transfer terms} \tag{16}$$

where the optical escape factor is defined as:

$$\Lambda(t_o) = 1 - \frac{4p\ n_1\ B_{12}}{n_2 A_{21}\ c}\int \bar{I}(n)\,f(n)\,dn \tag{17}$$

Using the average intensity obtained in equation (13), the optical escape factor can be written as:

$$\Lambda(t_o) = 1 - \frac{4p\ n_1\ B_{12}}{n_2 A_{21}\ c}\int_0^{\infty} \frac{c\ n_2\ A_{21}}{4p\ n_1\ B_{12}}\left[1 - \exp(-c(n)\,r\ \overline{D})\right]\ f(n)\,dn \tag{18}$$

$$\Lambda(t_o) = 1 - \int_0^{\infty} \left[1 - \exp(-c(n)\,r\ \overline{D})\right]\ f(n)\,dn \tag{19}$$

Substituting the *f(n)* function as defined in equation (19) we obtain:

$$\Lambda(t_o) = 1 - \frac{1}{\sqrt{p}\ \Delta n_D}\int_0^{\infty} \left[1-\exp(-c(n)\, r\ \overline{D})\right]\ \exp\left[\frac{-(n-n_o)^2}{\Delta n_D{}^2}\right] dn \qquad (20)$$

Now since

$$t(n) = \ c(n)\, r\ \overline{D} \qquad (21)$$

and

$$c(n) = \ \frac{n_1\ B_{12}\, f(n)\ \ hn}{r\ \ c} \qquad (22)$$

we have:

$$t(n) = \ \frac{n_1\ B_{12}\, f(n)\ \ hn}{c}\ \ \overline{D} \qquad (23)$$

Substituting the *f(n)* function once more:

$$t(n) = \ \frac{n_1\ B_{12}\ \ hn}{c\sqrt{p}\ \ \Delta n_D}\ \ \overline{D}\ \ \exp\left[\frac{-(n-n_o)^2}{\Delta n_D{}^2}\right] \qquad (24)$$

At $\nu = \nu_o$ we have:

$$t_o = \ t(n_o) = \ \frac{n_1\ B_{12}\ \ hn_o}{c\sqrt{p}\ \ \Delta n_D}\overline{D} \qquad (25)$$

$$t(n) = \ t_o \exp\left[\frac{-(n-n_o)^2}{\Delta n_D{}^2}\right] \qquad (26)$$

where τ_o is the mean optical depth of the plasma. Replacing in equation (20) we have:

$$\Lambda(t_o) = 1 - \frac{1}{\sqrt{p}\ \Delta n_D}\int_0^{\infty} \left(1-\exp\left\{\ t_o\ \ \exp\left[\frac{-(n-n_o)^2}{\Delta n_D{}^2}\right]\right\}\right)\ \exp\left[\frac{-(n-n_o)^2}{\Delta n_D{}^2}\right] dn \qquad (27)$$

Change of variable:

$$x \ = \ \frac{(n-n_o)}{\Delta n_D} \qquad\qquad dx \ = \ \frac{dn}{\Delta n_D} \qquad (28)$$

$$\Lambda(t_o) = 1 - \frac{1}{\sqrt{p}}\int_{-\infty}^{\infty} \left(\ 1-\exp\left\{\ -\ t_o \exp\ (x^2)\right\}\ \right)\ \exp\ (x^2)\ \ dx \qquad (29)$$

This integral has been solved by Mitchell and Zemansky [4]:

$$\Lambda(t_o) = 1\ \ -\left(\frac{t_o}{\sqrt{2}} - \frac{t_o{}^2}{\sqrt{3}\ \ 2!} + \frac{t_o{}^3}{\sqrt{4}\ \ 3!}\ \ \cdots\cdots\cdots \frac{(-1)^n\, t_o{}^n}{\sqrt{n+1}\ \ n!}\right) \qquad (30)$$

For large τ_o values, Drawin [5] has shown that:

$$\Lambda_o = \frac{1}{\tau_o \, (\pi \ \ln \ \tau_o)^{1/2}} \tag{31}$$

which is 1.6 times smaller than the value obtained by Holstein in a different calculation [6, 7]. Starting from equation (25) we can re-write τ_o in terms of observable quantities. First by replacing the $\Delta\nu_D$ value defined earlier:

$$\tau_o = \tau(\nu_o) = \frac{n_1 \ B_{12} \ h}{\sqrt{\pi} \ v_{th}} \overline{\Delta} \tag{32}$$

Second by utilizing the relationship between the different transition probabilities:

$$B_{12} = \frac{g_2}{g_1} B_{21} = \frac{g_2}{g_1} \frac{A_{21} \ c^3}{8 \ \pi \ h \nu_o^3} \tag{33}$$

In terms of wavelength the expression simplifies to:

$$\tau_o = \tau(\nu_o) = \frac{n_1 \ g_2 \ A_{21} \ \lambda_o^3}{8 \ g_1 \pi^{3/2} \ v_{th}} \overline{\Delta} \tag{34}$$

The Optical Escape Factor Λ_o can now be calculated by first using equation (34) to obtain the Mean Optical Depth τ_o and, then equation (30). The OEF values are shown in table 1 and in figure (1) as a function of the MOD. As mentioned earlier, OEF is close to unity for MOD values smaller than 0.01.

5.0 Evaluation of Optical Escape Factors and Mean Optical Depths for several He transitions

5.1 Evaluation of the lower excited level populations (n_1)

Some important transitions of the helium atom are shown in the Grotrian diagram (see figure 2). Of particular interest for a line ratio diagnostic [8] are the singlet transitions: $n^1S \rightarrow 2^1P$ at 443.8, 504.8, and 728.1 nm and, the triplet transitions: $n^3S \rightarrow 2^3P$ at 412.0, 471.3 and 706.5 nm. According to equation (34) the Mean Optical Depth depends on the lower excited level not on the population of the upper level. In order to evaluate the lower excited populations (2^1P and 2^3P), we must first evaluate the neutral population in the plasma column. We consider the average plasma condition with a filling pressure of 10 mTorr (real pressure). At this pressure, the electron temperature is about 10 eV (Langmuir Probe) and the average neutral temperature within the plasma (measured by LIF) is about 0.05 eV (580 ^oK) [9]. Thus, the neutral density in plasma column is about 1.7 x 10^{14} cm^{-3}. The average plasma radius is about 1.5 cm. We must then use a model to predict what fraction of the neutral population is within a given excited state. For neutral density lower than 10^{11} cm^{-3}, the simpler steady coronal model can be used, while for higher density ($10^{11} < n_e \leq 10^{14}$ cm^{-3}) the collisional radiative model is a better choice. To be consistent, we used Broda's collisional radiative model [10] to evaluate all the population of the lower excited levels involved in the selected transitions. In this work, all the transitions ended in the 2^1P and the 2^3P levels. Using T_e = 10 eV and figures (3) and (4) from Broda [10], the population of the 2^1P and 2^3P excited levels are obtained. These population numbers are shown in tables 2 and 3.

5.2 Evaluation of the Optical Escape factors and Mean Optical Depth

The statistical weights g_1 and g_2, the wavelengths and the transition probabilities A_{ji} are given in tables [11]. The neutral thermal velocity is obtained through the LIF measurement [9]. Using equation (34) and equation (30), the Mean Optical Depths and Optical Escape Factors are calculated for the He transitions listed earlier. The MOD and OEF are shown in tables 2 to 7. A similar calculation can be used to evaluate the MOD and OEF for the remaining He transitions not listed here. Is the plasma condition used for the calculation in section 5.1 representative of all optical situations that can be found in HELIX plasmas? Yes, other plasma conditions will yield similar MOD and OEF coefficients. A thinner plasma (n_{He} smaller) will yield comparable n_1 population since T_e is expected to be larger for this condition. A high temperature, high density plasma (high power discharge) will induce a large n_1 population but will also generates a large v_{th} values and so on. In all cases, the different T_e, n_1 and v_{th} values acts as check and balance factors in equation (34) resulting in MOD values comparable to those listed in tables 2 to 7.

6.0 Interpretation

For all singlet transitions, the OEF is essentially unity even for electron density up to 10^{14} cm^{-3}. This means that the plasma is essentially thin with respect to these singlet transitions. For the triplet transitions, the situation is somewhat less favorable. Essentially, transitions from the upper levels 5^3S and 4^3S (412.0 and 471.3 nm, respectively) can be considered as optically thin for plasma with electron density up to 10^{14} cm^{-3} (less than 2% of re-absorption in each case). Meanwhile, the transition from the 3^3S level (706.5nm) is largely re-absorbed by the plasma (≈ 5% at 10^{11} cm^{-3}, ≈ 40% at 10^{14} cm^{-3}). Here, a strong transition (large A_{ji}), a large n_1 population (the presence of the long-lived 2^3S metastable near the 2^3P level is essentially responsible for this large n_1 population) and, a long wavelength ($\tau_o \propto \lambda_o^3$) all contribute to make the MOD large enough (0.065 ($10^{11}cm^{-3}$) < τ_o < 0.72 (10^{14} cm^{-3})) so that the plasma is no longer transparent to this transition. Thus, the 706.5 nm line intensity must be corrected for re-absorption. Since the re-absorption is function of the neutral density profile (supposed uniform in this calculation), the correction is not a simple operation. If possible, the line ratio diagnostic should not use this transition.

7.0 Conclusion

We have shown how to evaluate the Optical Escape Factor and the Mean Optical Depth for He transitions in plasmas. This calculation reveals that most transitions are optically thin with respect to the plasma for electron density up to 10^{14} cm^{-3}. This is in part due to the small relative numbers of the lower excited populations. Provide the He transition does not involved the ground state or any of the 2^1S, 2^3S or sometimes 2^3P levels, the transition is optically thin with respect to the plasma for electron density up to 10^{14} cm^{-3}. For resonance transition, the MOD and OEF calculations are necessary since these transitions will be re-absorbed by the plasma. The method could be extended to other gases provide a collisional radiative model can be obtained to evaluate the lower excited populations.

8.0 References

[1] Boivin R. F., Spectroscopy System for the WVU Helicon Plasma Devices, Internal Report WV-PL- 046, West Virginia University (2000)

[2] McWhirter R. W. P. in Plasma Diagnostic Techniques, Chapter 5, R. H. Huddlestone and S. L. Leonard, Academic Press, (1965)

[3] Boivin R. F., Study of the Different Line Broadening Mechanism for the Laser Fluorescence Diagnostic for the HELIX and LEIA Plasmas, WV-PL-039, West Virginia University, (1998)

[4] Mitchell A. C. and M. W. Zemansky, Resonance Radiation and Excited Atoms, Cambridge Press, (1971)

[5] Drawin H. W. and F. Emard, Beitr. Plasmaphysik **30a**, 143 (1973)

[6] Holstein T., Phys. Rev. **72**, 1212 (1947)

[7] Holstein T., Phys. Rev. **83**, 1159 (1951)

[8] Boivin R. F., J. L. Kline and, E. E. Scime, Electron Temperature Measurements in Helicon Plasmas by Helium Line Intensity Ratios, To be submitted to Physics of Plasmas (2001)

[9] Scime, E. E., P. A. Keiter, M. W. Zintl, M. M. Balkey, J. L. Kline, and M. E. Koepke, Plasma Sources Sci. Technol. **7**, 186 (1998)

[10] Brosda B. Ph.D. Thesis, Ruhr-Universitat, Bochum, (1993)

[11] Wiese W. L., M. W. Smith, and B. M. Glennon, Atomic Transition Probabilities, Vol.1. National Standard Reference Data Series (1966)

Table 1. Numerical values for the Mean Optical Depth and Optical Escape Factors (36 terms in the summation, see equation (30))

τ_o (Optical Depth)	Λ (Escape Factor)
1E-6	1.0
5E-6	1.0
1E-5	1.0
5E-5	1.0
1E-4	0.9999
5E-4	0.9997
1E-3	0.9993
5E-3	0.9965
1E-2	0.9930
5E-2	0.9654
1E-1	0.9321
5E-1	0.7093
1.00	0.5139
5.00	0.0895
10.00	0.0405

Table 2 - 7

Lower level excited population (n_1), Mean Optical Depth (τ_o), attenuation fraction (I/I_o) and Optical Escape Factor (Λ) for the different n^1S ➔ 2^1P and n^3S ➔ 2^3P ($n \geq 3$) transitions as a function of plasma density.

Table 2. Mean Optical depth and optical escape factor for the He I 412.0 nm transition

n_e (cm^{-3})	n_1(cm^{-3})	τ_o	I/I_o	Λ
10^{11}	6.52×10^9	1.85×10^{-3}	.998	.9987
10^{12}	5.10×10^{10}	1.45×10^{-2}	.986	.9898
10^{13}	1.05×10^{11}	3.00×10^{-2}	.970	.9790
10^{14}	7.93×10^{10}	2.25×10^{-2}	.978	.9841

Table 3. Mean Optical depth and optical escape factor for the He I 443.8 nm transition

$n_e\,(cm^{-3})$	$n_1(cm^{-3})$	τ_o	I/I_o	Λ
10^{11}	7.09×10^6	1.79×10^{-6}	1.00	1.000
10^{12}	6.80×10^7	1.72×10^{-5}	1.00	1.000
10^{13}	6.80×10^8	1.72×10^{-4}	1.00	.9999
10^{14}	5.95×10^9	1.50×10^{-3}	.999	.9989

Table 4. Mean Optical depth and optical escape factor for the He I 471.3 nm transition

$n_e\,(cm^{-3})$	$n_1(cm^{-3})$	τ_o	I/I_o	Λ
10^{11}	6.52×10^9	2.00×10^{-3}	.998	.9986
10^{12}	5.10×10^{10}	1.55×10^{-2}	.985	.9890
10^{13}	1.05×10^{11}	3.20×10^{-2}	.969	.9777
10^{14}	7.93×10^{10}	2.45×10^{-2}	.976	.9825

Table 5. Mean Optical depth and optical escape factor for the He I 504.8 nm transition

$n_e\,(cm^{-3})$	$n_1(cm^{-3})$	τ_o	I/I_o	Λ
10^{11}	7.09×10^6	1.90×10^{-6}	1.00	1.000
10^{12}	6.80×10^7	1.80×10^{-5}	1.00	1.000
10^{13}	6.80×10^8	1.80×10^{-4}	1.00	.9999
10^{14}	5.95×10^9	1.60×10^{-3}	.998	.9989

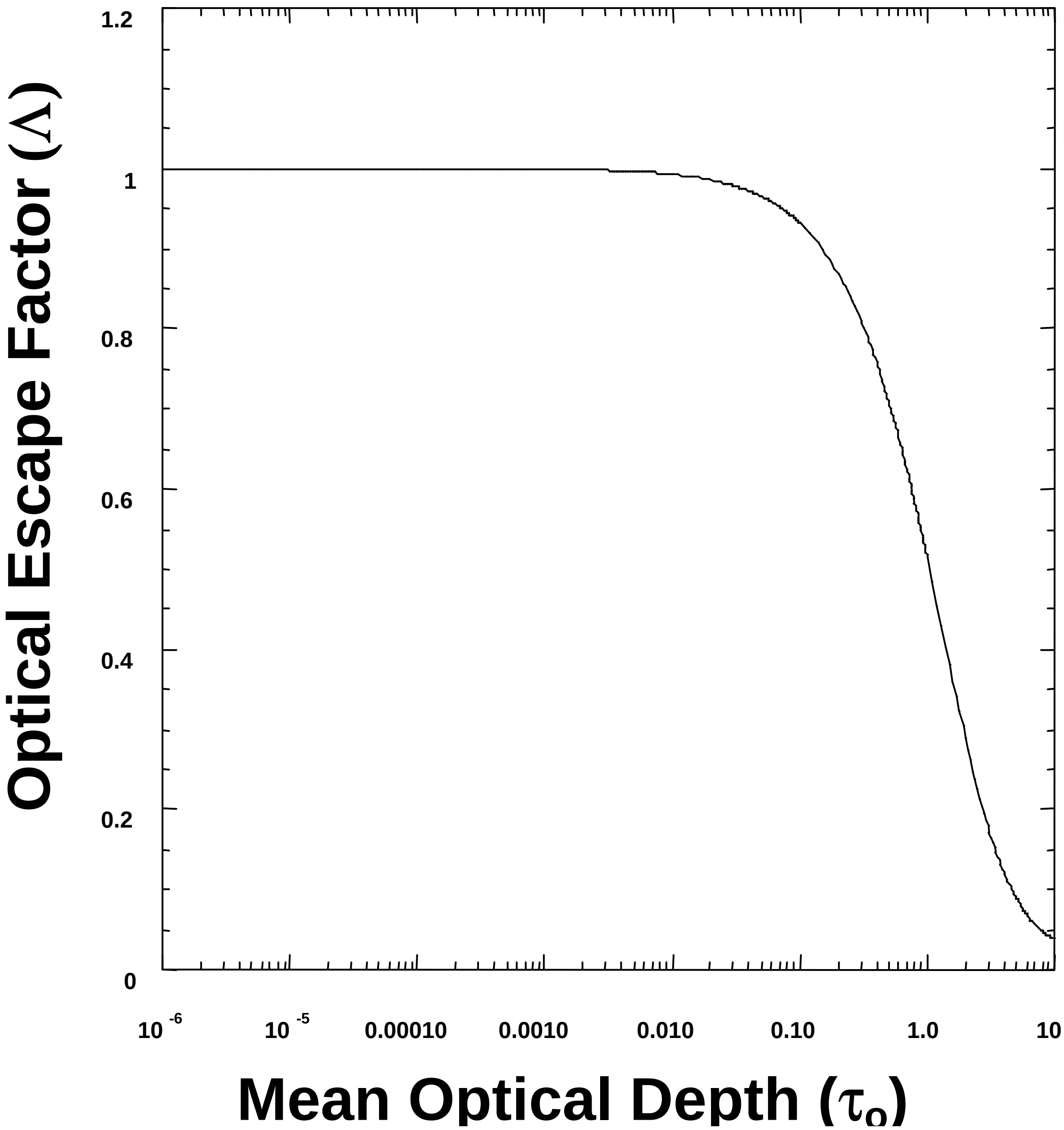
Table 6. Mean Optical depth and optical escape factor for the He I 706.5 nm transition

$n_e\,(cm^{-3})$	$n_1(cm^{-3})$	τ_o	I/I_o	Λ
10^{11}	6.52×10^9	6.65×10^{-2}	.936	.9542
10^{12}	5.10×10^{10}	4.61×10^{-1}	.631	.7285
10^{13}	1.05×10^{11}	8.45×10^{-1}	.389	.5666
10^{14}	7.93×10^{10}	7.15×10^{-1}	.489	.6158

Table 7. Mean Optical depth and optical escape factor for the He I 728.1 nm transition

$n_e\,(cm^{-3})$	$n_1(cm^{-3})$	τ_o	I/I_o	Λ
10^{11}	7.09×10^6	4.55×10^{-5}	1.00	1.000
10^{12}	6.80×10^7	4.40×10^{-4}	1.00	.9997
10^{13}	6.80×10^8	4.40×10^{-3}	.996	.9969
10^{14}	5.95×10^9	3.85×10^{-2}	.962	.9733

Fig. 1. Optical Escape Factor as a function of the Mean Optical Depth



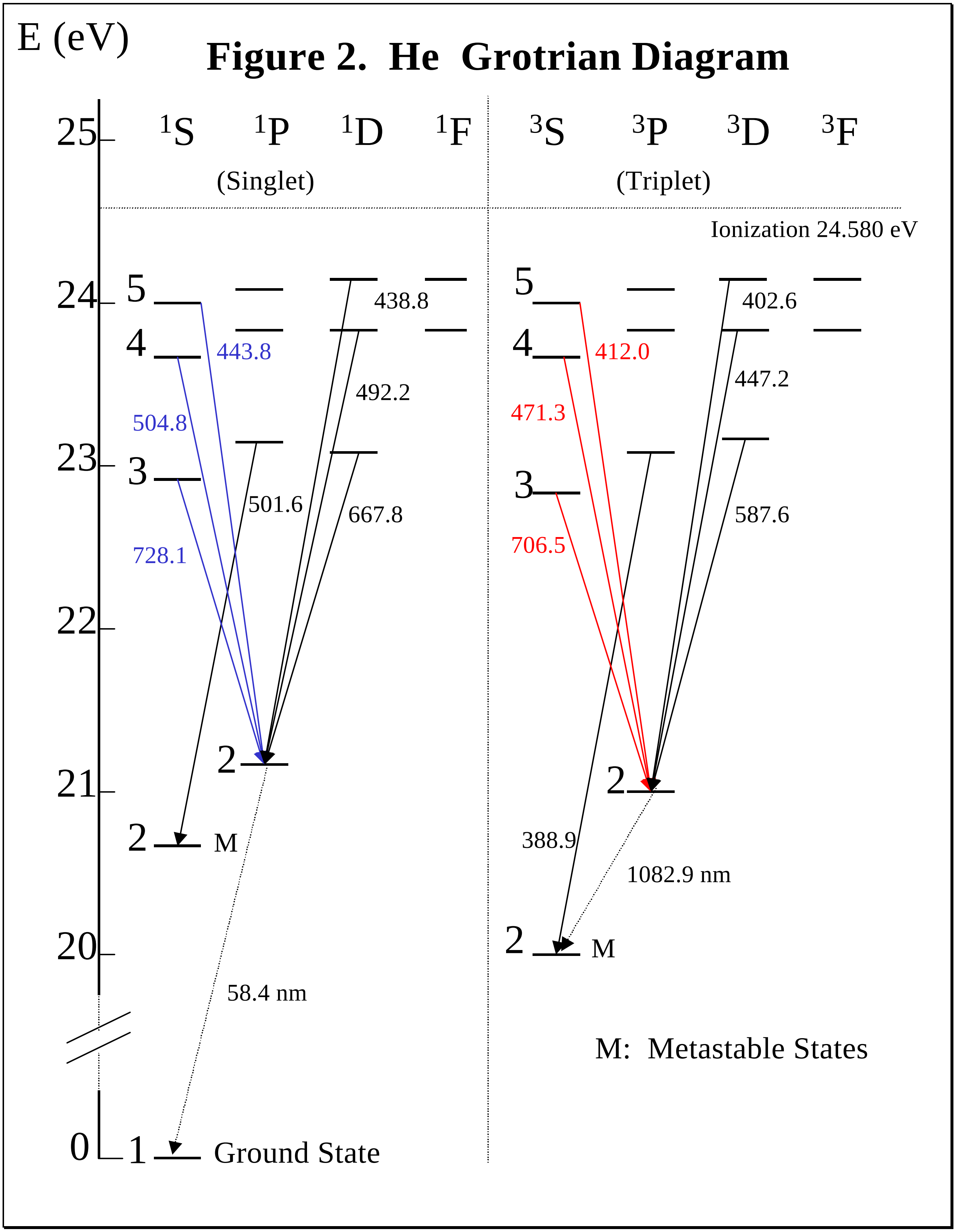


Figure 2. He Grotrian Diagram

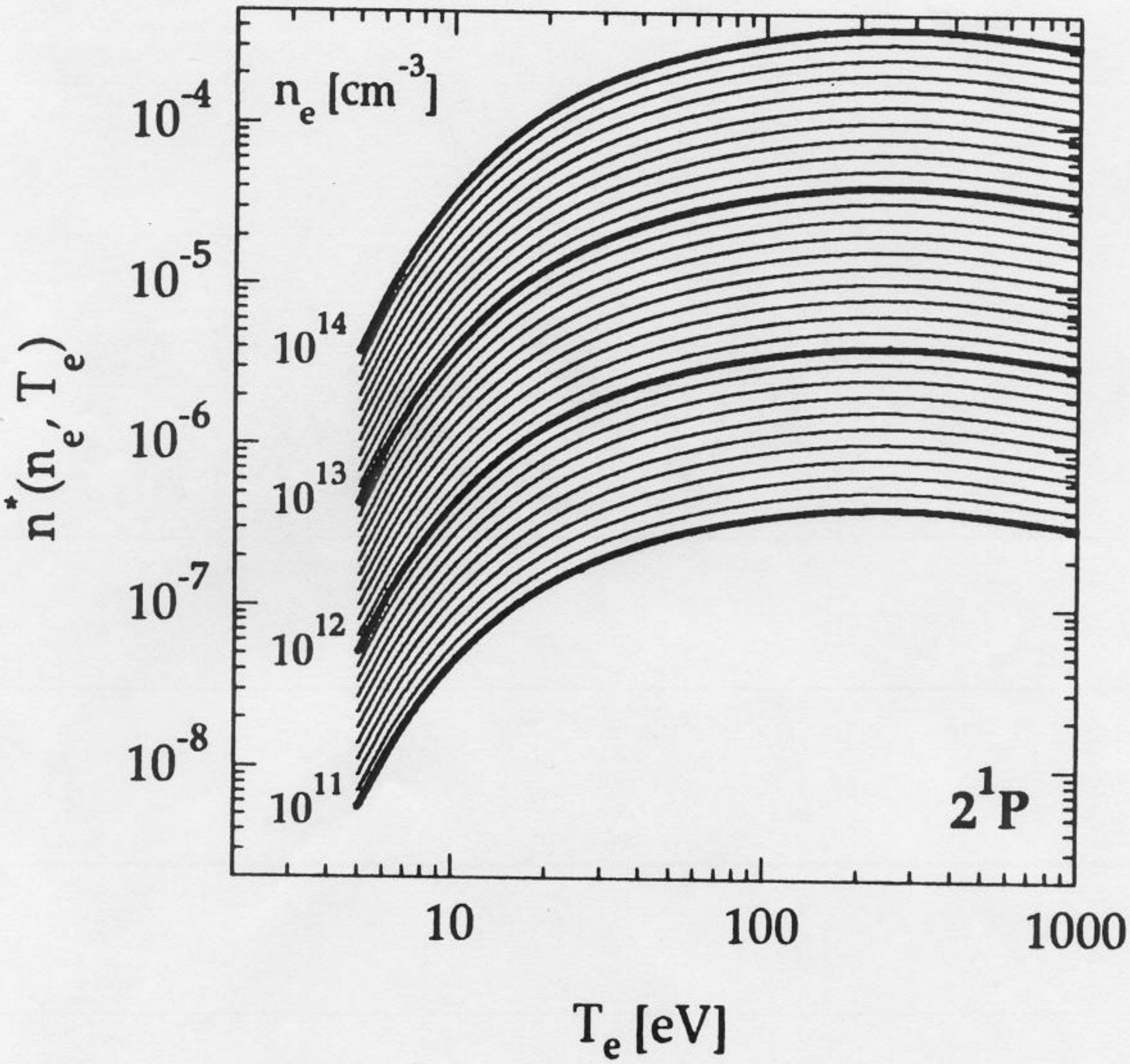


Figure 3. Population of the 2^1P level according to the Collisional Radiative Model [10] as a function of the electron temperature and for different plasma density.

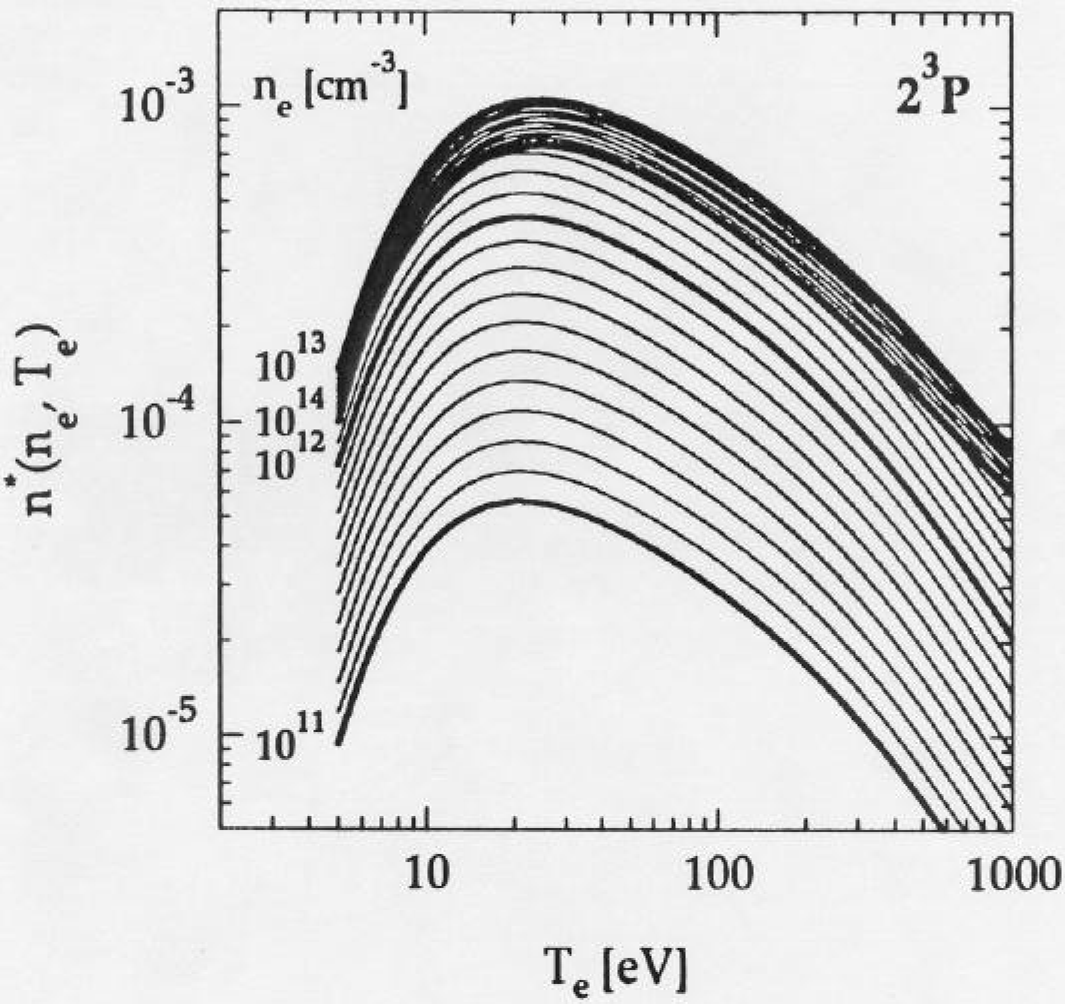


Figure 4. Population of the 2^3P level according to the Collisional Radiative Model [10] as a function of the electron temperature and for different plasma density.